Exit d'or

08

Lionel March
University of California, Los Angeles, USA

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Lionel March
Professor of Design and Computation, Center for Medieval and Renaissance Studies, University of California, Los Angeles, CA 90095-1615 USA

Abstract

The *section d'or* has held an all too revered a place in proportional design. Egyptian monuments, Greek temples and all manner of architectural works since have been 'shown' to follow the inexorable geometry of the 'divine' proportion. This paper does not review the history of the number commonly referred to in design circles as Φ, phi. That has been done well elsewhere.[1] Instead it takes two examples in which the *section d'or* is said to have determined the design: Leonardo da Vinci's famous Vitruvian man, and the Villa Emo of Palladio. In both cases, clearly marked dimensions on the drawings show that the *section d'or* was emphatically not used.

Leonardo da Vinci: the Vitruvian man

Leonardo da Vinci pictorially interprets this passage in Vitruvius 3.1.3:[2]

> ... the center and midpoint of the human body is, naturally, the navel. For if a person is imagined lying back with outstretched arms and feet within a circle whose center is the navel, the fingers and toes will trace the circumference of this circle as they move about. But to what ever extent a circular scheme may be present in the body, a square design may also be discerned there. For if we measure from the soles of the feet to the crown of the head, and this measurement is compared with that of outstretched hands, one discovers that this breadth equals the height just as in areas that have been squared off by use of the set square.

With a familiar graphic construction, Robert Lawlor[3] in *Sacred Geometry* overlays a reproduction of the drawing, Figure 1. In a book in which the *section d'or* takes pride of place in nature and in design, it comes as no surprise to find that Lawlor demonstrates that Leonardo's Vitruvian figure conforms to this rule of 'universal harmony'. Indeed, addressing the prurient adolescent in many of us, Lawlor writes:

> The body is divided exactly in half by the sex organs. This denotes the relationship of sexuality with the dualizing function, the division into two. At birth, however, it is the *navel* that divides the child exactly in half, and in the course of maturation the navel moves to the point of the phi division. Thus the position of the navel through human growth is related to the idea of movement from the dualized, sexualized stance in nature to that of a proportional relation to Unity through the asymmetrical, dynamic power of Φ.

The drawing appears to be convincing. But I am sceptical of its relevance. It is true that Luca Pacioli[4] used Leonardo's Latin alphabet and his drawings of polyhedra in *De divina proportione*, and that the first section of that book identified the extreme and mean ratio — the *section d'or* — with traits of the Christian godhead. At no point, however, does Pacioli give this ratio the unique aesthetic status it was eventually to acquire in and from the 19th century. In the second section, Pacioli summarizes his understanding of theVitruvian code, without a whisper on the extreme and mean ratio this Franciscan friar has so loudly promoted on theological grounds in the first section. Unless there is an occult play on the theme of man and divinity, which is certainly a possibility, it seems to me unlikely that Leonardo would have employed this ratio in theVitruvian context of his drawing given Pacioli's aesthetic disinterest in such a relationship.

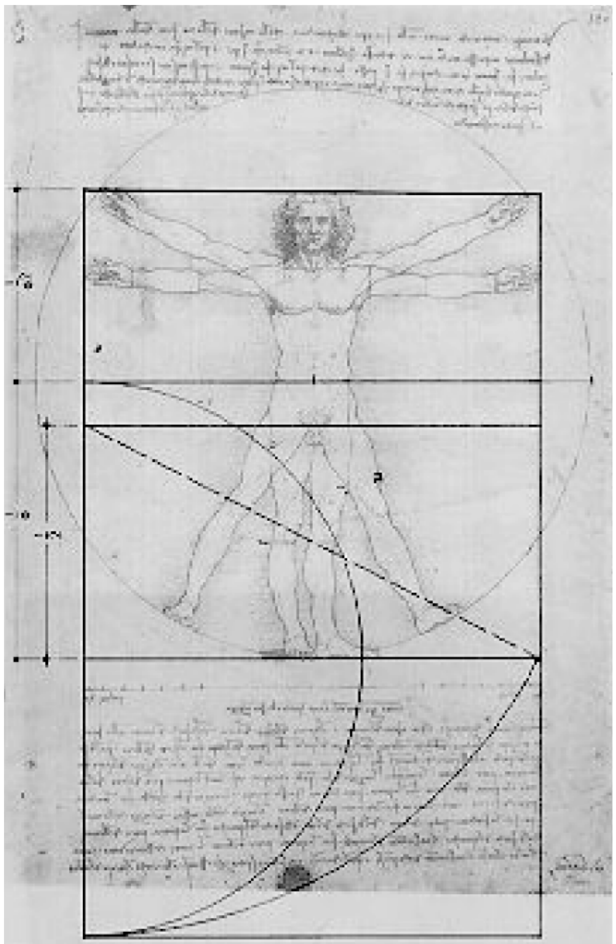

Figure 1. Robert Lawlor's *section d'or* analysis of Leonardo's Vitruvian man.

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Is there an alternative? Leonardo describes the Vitruvian man in two passages. First, the general disposition:[5]

> If you open your legs so that you lower your head by one-fourteenth of your height, and open and raise your arms so that with your longest fingers you touch the level of the top of your head, you should know that the central point between the extremities of the outstretched limbs [feet and hands] will be the navel. and the space which is described by the legs makes an equilateral triangle.

In the other passage, Leonardo explains his choice of measurements for the scale he draws at the base of the figure:

> Vitruvius, the architect, has it in his work on architecture that the measurements of man are arranged by nature in the following manner: four fingers make one palm and four palms make one foot; six palms make one cubit; four cubits make a man, anf four cubits make one pace; twenty-four palms make a man; and these measurements are those of his buildings.

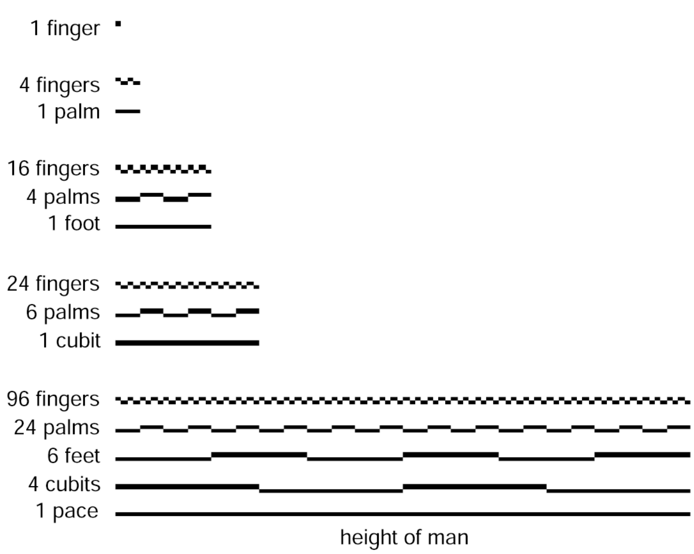


Figure 2. Leonardo's measurements.

The system of Roman measurements is shown in Figure 2.

The adjustment of the man's height by one fourteenth when his legs are parted is another matter. First it is necessary to examine the relationship of the square to the circle. Using Leonardo's own scale it is evident that the square has sides of length 24 palms. With the exact same scale, the diameter of the circle is found to be 29 palms, Figure 3.

Consider Table 1 where successive rational convergents (approximations) to the ratio √2:1 are given:[6]

√2:1 1:1 2:1 3:2 4:3 7:5 10:7 17:12 24:17 41:29 ...

The earliest values are poor, at the very start just plain bad, but they improve rapidly. The values 7 : 5 and 10 : 7 were commonly used by the Roman builders in setting out rooms. Alberti[7] does not consider 7 : 5 to be a particularly good value. The better value 17 : 12 is explicitly illustrated in Cesariano's 1521 edition of Vitruvius, after, of course, Leonardo's death. Yet Cesariano is surely reporting on a well known estimate. It appears that Leonardo learned about roots from Luca Pacioli: this most likely meant knowing about algorithms to extract rational convergents for roots. David Fowler[8] has covered this classical tradition: a tradition which persists through the middle ages, into the renaissance of Leonardo's time and beyond.

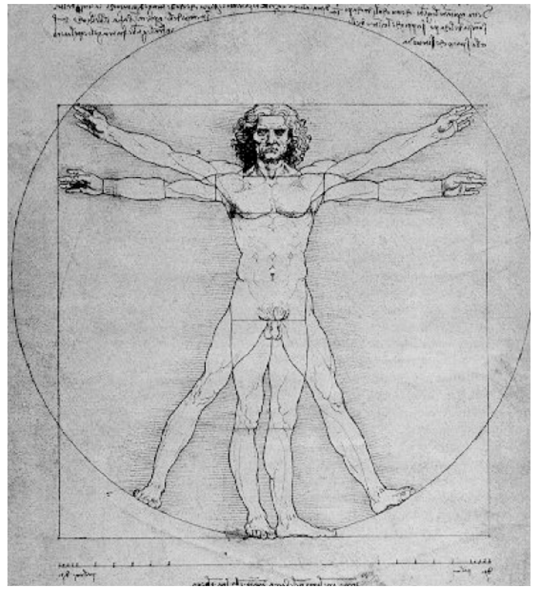


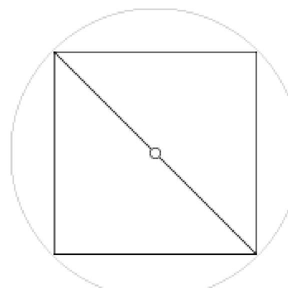
Figure 3. Leonardo's Vitruvian man with his own scale in palms superimposed.

The numbers 24 and 29 appear in the sequence in Table 1 and alert us to the possibility that, in his Vitruvian man, Leonardo is toying with geometry associated with shapes related to the right isosceles triangle (half-square cut along the diameter), Figure 4a.

Apart from the square itself, one such figure is the regular octagon which Leonardo was fond of using — almost to exclusion —particularly in his many proposals for centralized churches. This is certainly the polygon that Leonardo used as the underlying geometry of his Vitruvian illustration. It appears that Leonardo employed the rational convergent 17 : 12 for the diameter and sides of a square, Figure 4b.

Consider a square of 24 palms, both the height of a man and his span with arms outstretched. A regular octagon whose sides are half those of this square, 12 palms, circumscribes a circle of diameter 29 palms, Figure 5.

This firmly establishes the relationship between the square and the circle in Leonardo's construction. With respect to the figure when the legs are parted, it appears that Leonardo usesthe Milan cathedral formula[9] for the height to width of an equilateral triangle, that is, with an altitude of 7, the side lengths of the triangle are each very close to 8. The formula may



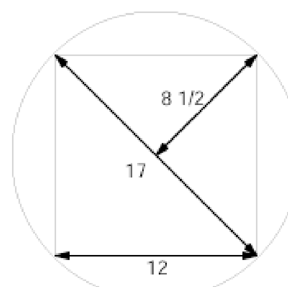


Figure 4a above. The diameter of the square to the side is in the ratio 2 : 1.
Figure 4b below. A square marked with rational values for the side, diameter and radius.

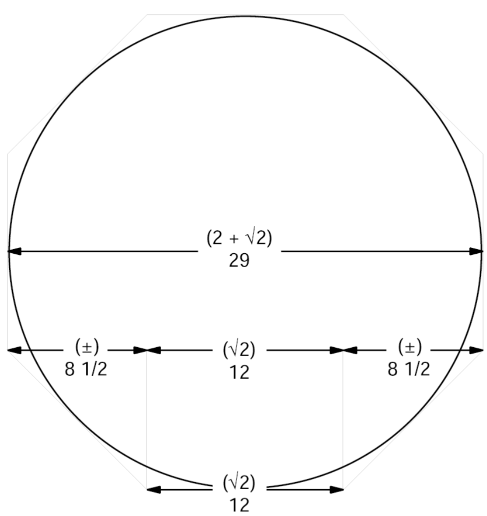


Figure 5. left. A circle of diameter 29 is inscribed in a regular octagon of side lengths 12.

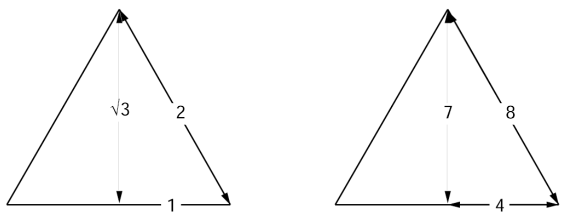


Figure 6. The Milan Cathedral rational approximation for an equilateral triangle.

be tested by Pythagoras' theorem. The half equilateral triangle is a right-angled scalene with exact dimensions of 1, $\sqrt{3}$, 2. The rational approximation used at Milan gives a triangle with dimensions 4, 7, 8. Pythagoras' theorem provides the values:

$4^2 + 7^2 = 16 + 49 = 65,$
$8^2 = 64.$

It can be seen that the expected equality is close, Figure 6.

Locating the equilateral triangle in the circle is accomplished by inscribing a regular hexagon, Figure 7.

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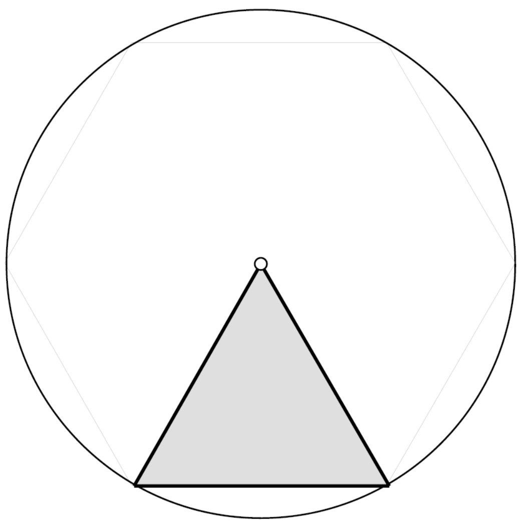
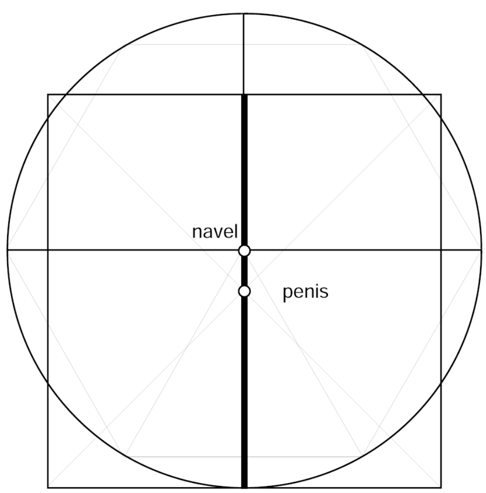




When the 24 x 24 square is added to the figure so that its base is tangential to the bottom of the circle, the extent of the standing man, feet together and arms horizontally outstretched, is defined. The center of the square is the penis. The apex of the equilateral triangle locates the navel. It will be seen that the altitude of the equilateral triangle is close to the half-side of the square. If the Milan Cathedral 8 : 7 ratio is assumed for the triangle, the radius of the circle will be 8 units and the distance between where the feet were when they were together to where they are when apart is 8 - 7 = 1 unit. The original height of the man in these units is 7 + 7 = 14. Hence, the shortening by 1/14th to which Leonardo refers, Figure 8.

To impose the doctrine of the *section d'or* on Leonardo's defining icon of humanistic endeavour is truly an outrage. Historically, it is anachronistic. Such a doctrine did not figure prominently in Leonardo's, nor his contemporaries, aesthetic criteria. If it ever did, it was most probably more by accident than design. What did fascinate the times were the regular figures in two and three dimensions, figures exhibiting strong central symmetry. Wittkower[10] discusses religio-cultural reasons for this enthusiasm for centrality in architecture. The mathematical interest should not be ignored. From 1496, when he met Luca Pacioli, Leonardo spent much time dabbling with 'rationally' intractable geometrical problems such as the quadrature of the circle and the doubling of the cube — all to no avail mathematically ... but the doodles!. Noting his total absorption in mathematical matters during this period, an observer in 1501, writes of Leonardo: "the sight of a brush puts him out of temper".[11]

Figure 7. above. An equilateral triangle inscribed in the circle

Figure 8. below. Diagram showing the reduction in height by 1/14th when the legs are parted to align with the sides of of an equilateral triangle concurrent with the navel.

It is known that Leonardo was familiar with the 1512 printed edition of Alberti's *Libri de re aedificatoria decem* ... (Paris). This would be seven years before his death. He may have had knowledge of Alberti's work from the earlier 1486 edition (Florence). In any event, it is intruiging to read in Book 7 the passage concerning many-sided architectural plans.[12] Having written that "Nature delights primarily in the circle", Alberti continues:

> ... the ancients would use six, eight, or even ten angles. The corners of all such plans must be circumscribed by a circle. Furthermore, they may be plotted exactly using the circle. For half the diameter of the circle will give the length of the sides of the hexagon. And if you draw a straight line from the center to bisect each of the sides of the hexagon, it is obvious how to construct the dodecagon. From a dodecagon it is obvious how to derive an octagon, or even a quadrilateral.

The lineaments of Leonardo's drawing are strikingly based on the overlaid geometries of such regular figures. All of them may be derived from the regular dodecagon — the octagon, the hexagon, the square and the triangle. It is as if Leonardo extracts them all from the primary matrix of the dodecagon, Figure 9.

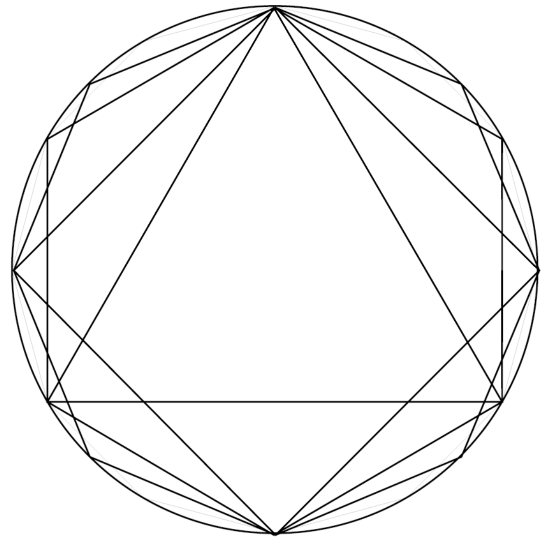


Figure 9. The nesting of regular polyhedra in a circle

Leonardo does not here employ the pentagon, nor the decagon, both of which would have introduced the *section d'o*r. It is hard to find in all his copious works an interest in these two figures. It indicates that the use of this unique ratio was in no way a priority for Leonardo. Instead, he flourished in the abundance of proportional schemes that geometrical configurations generate when variety is favored over uniformity; opposition over sameness.

Palladio's Villa Emo

In a most thoughtful and persuasive paper[13], Rachel Fletcher comes close to convincing that Palladio may well have made use of the *section d'o*r, or extreme and mean ratio, in the design of the Villa Emo at Fanzolo which was probably conceived and built during the decade 1555-1565. It is early in this period, 1556, that *I dieci libri dell'archittetura di M. Vitruvio Pollionis traduitti et commentati* ... by Daniele Barbaro was published and the collaboration of Palladio acknowledged[14]. In the later Latin edition of 1567[15], there are geometrical diagrams of the equilateral triangle, square and hexagon which evoke ratios involving $\sqrt{2}$ and $\sqrt{3}$, but there are no specific drawings of pentagons, or decagons, which might explicitly alert the perceptive reader to the extreme and mean proportion,

$$1 : \phi :: \phi : \phi^2.$$

Architectural examples employing $\sqrt{2}$ and $\sqrt{3}$ include the Roman theater and Greek theater, respectively. The most telling use of the pentagon occurs as a minor detail in two *inventioni* for architraves surrounding doors and windows[16] — but more of this later. How would Barbaro — and perhaps his illustrator, Palladio — have constructed a pentagon or decagon? In the mid-fifteenth century, Alberti had described in words an exact construction for the decagon[17]. Albrecht Duerer, 1525, illustrates two distinct constructions for the pentagon, one according to geometric theory, and another traditionally used by masons and craftsmen which is only approximate[18]. By the 1540s, Serlio shows Duerer's exact construction[19]; yet as late as 1569, Barbaro shows only Duerer's approximate construction[20]. Whereas the exact construction leads to the extreme and mean ratio, the approximate construction does not. Someone seriously aware of the relationship of the extreme and mean ratio to the pentagon, or decagon, would surely use the exact method, especially if that relationship was seen to have aesthetic value. But there really is no evidence that any of these authors had strong commitments to the extreme and mean ratio for aesthetic purposes.

It is Kepler in the seventeenth century who connects the extreme and mean ratio with natural phenomena such as planetary motion, and makes the discovery that successive pairs in the sequence 1, 2, 3, 5, 8, 13, ... converge on the value of the extreme and mean ratio — without in anyway relating this to the sequence which occurs in a problem solved by Fibonacci in the thirteenth century and had laid dormant until its rediscovery in the nineteenth[21]. The extreme and mean ratio emerges, born again as the *section d'o*r, as a key to aesthetic measure only in the nineteenth and twentieth centuries. Over the last century and a half, its aesthetic use has been sanctioned, even sanctified, by casting its diagrammatic aura over the analysis of past works in the arts from architecture, to painting and sculpture, to music and poetry; and by observing its pervasive presence in nature, in growth patterns, or phyllotaxis[22]. None of this will be found in renaissance commentaries. None.

It is true that the Fibonacci ratios 1 : 1, 2 : 1, 3: 2, 5 : 3, 8 : 5 , 13 : 8 will be found in Palladio's works, but they represent less than six per cent of all ninety ratios to be found in Book II , nor do they occur as a coherent set in any, but one, work[23]. Except for 13 : 8, the remaining five ratios have a musical interpretation within the contemporary *senario* of the music theorist Gioseffo Zarlino[24]. However, the ratio 13 : 8 produces an interval which is very much out of tune with the modern major and minor scales then beginning to displace the traditional modes , while $\phi : 1$ itself is yet more cacophonic and utterly disharmonious in musical theory and to the ears.

Palladio does use ratios which better converge towards the finitely unreachable extreme and mean ratio. These lie between the underestimate 8 : 5 [1.6] and the overestimate 5 : 3[1.66667]. The ratio 13 : 8 [1.625] is among these, but 21 : 13 [1.615385] is not one of them. In his reconstruction of a private house for the ancient Romans[25], the atrium is shown with dimensions $83^1/_3$ by 50 piedes. The additional one third of a piede neatly turns this into the ratio 5 : 3. Of this ratio, Palladio writes: "I like very much those rooms which are two-thirds longer than their breadth"[26].

There is ample evidence that Palladio employed ratios related to regular planar figures such as those Leonardo da Vinci used[27]. To arrive at such proportional design, it seems that Palladio would have made use of rational estimates for square roots of non-square numbers, such as 2, 3 and 5. There were several techniques for computing the numerical values at the time, but once such computations were made it would probably have been convenient to look them up in tables, or simply to remember at least the most commonly used values. Typical values in the generative process which converge on these square root are given in Table 2[28]. Note that the ratio 5 : 3 may stand for $\sqrt{3} : 1$, and is not to be read uniquely as an early term in a Fibonacci approximation to $\phi : 1$.

Table 2. Rational convergents to the square roots of 2, 3 and 5. Ratios shown in bold occur in Palladio's Book II. The early values to the left are embryonic, those to the right are more mature.

√2 : 1	**1 : 1**	**3: 2**	**7 : 5**	**17 : 12**	41 : 29	99 : 70	...
	2 : 1	**4 : 3**	10 : 7	**24 : 17**	58 : 29	140 :99	...
√3 : 1	**1 : 1**	**2 : 1**	**5 : 3**	**7 : 4**	**19 : 11**	**26 : 15**	...
	3 : 1	**3 : 2**	**9 : 5**	**12 : 7**	33 : 19	45 : 26	...
√5 : 1	**2 : 1**	7 : 3	11 : 5	**9 : 4**	29 : 13	47 : 21	...
	5 : 2	15 : 7	**25 : 11**	**20 : 9**	65 : 29	105 :	...

Earlier, the discussion of pentagonal proportional design in two *inventioni* by Palladio was postponed. With Table 2 at hand, it is now possible to proceed. The designs are for door and window architraves. Palladio illustrates how to set out the *gola diritta*, an S-shaped moulding in the cornice[29]. Palladio describes the construction of this curve: "To make it well and gracefully, draw a straight line AB and divide it into two equal parts at the point C; divide one of these halves into seven parts and make six of these coincide at point D; then one forms two triangles AEC and CBF; and at the points E and F fix the compass and draw the segments of a circle AC and CN which form the *gola*", Figure 10.

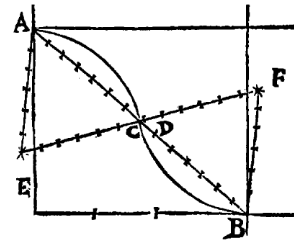
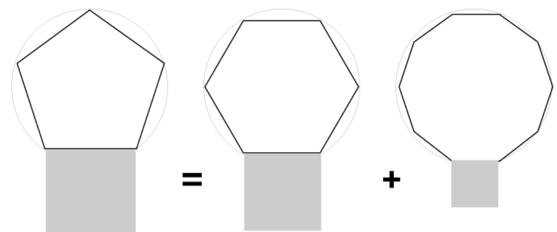


Figure 10. Above. Palladio's construction for setting out the *gola diritta* of a cornice.

This construction is no whim. It derives from an arithmetical interpretation of Euclid, Proposition 10, Book XIII[30]. Unquestionably to be counted among the most aesthetically pleasing of all the propositions in *The Elements*, Proposition 10 reads: "If an equilateral pentagon be inscribed in a circle, the square on the side of the pentagon is equal to the squares on the side of the hexagon and on that of the decagon inscribed in the same circle", Figure 11.

Figure 11. Below. Euclid's proposition which states that the square on the side of an equilateral pentagon is equal to the sum of the squares on the sides of the hexagon and the decagon.



Let the side of the pentagon be s, and the radius of the common circle be r, Figure 12.

The side of the hexagon is equal to the radius, and the side of the decagon is in proportion to the radius as 1 : ϕ. In modern terms, Proposition 10 may be expressed algebraically as:

$$s^2 = r^2 + \left(\frac{r}{\phi}\right)^2,$$

whence, the side of the pentagon

$$s = r\sqrt{1 + \frac{1}{\phi^2}}\ .$$

Using the defining relation

$$\phi^2 = \phi + 1,$$

and the value

$$\phi = \frac{1+\sqrt{5}}{2},$$

the expression for the side can be reduced to

$$s = \frac{r}{2}\sqrt{10 - 2\sqrt{5}}\ .$$

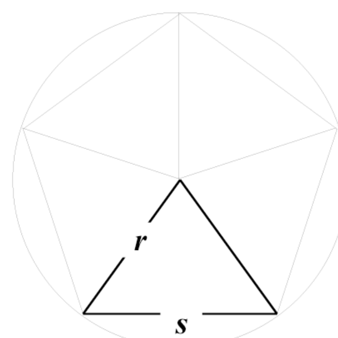


Figure 12. Equilateral pentagon.

Computationally, this is what Euclid's proposition implies; and, without the advantages of modern algebraic notation, this is very much the kind of procedure that Piero della Francesca would have had to follow in his fifteenth century programme for the arithmeticization of Euclidean geometry. How would such an expression, albeit in different notation, be evaluated? It would be necessary to substitute a rational value for $\sqrt{5}$. But what value? It would be convenient, if the remaining square root after the substitution was of a square, or near-square, number. Scanning through Table 2, the values 9/4 and 20/9 show promise since the numerators are square numbers and their roots can be brought outside the main square root sign. The value 9/4 leads to

$$\frac{r}{4}\sqrt{22},$$

but 22 is not a near-square number, whereas the value 20/9 gives

$$\frac{1}{6}\sqrt{50}\ ,$$

and $\sqrt{50}$ is very close to $\sqrt{49} = 7$. Thus, a good rational solution , shown in Figure 13, is:

$$s : r :: 7 : 6\ .$$

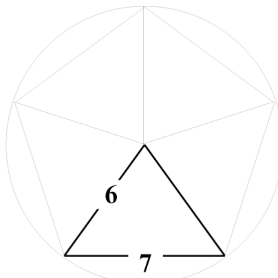


Figure 13. Natural numbers assigned to the radius and side of the equilateral pentagon.

This is precisely the ratio that Palladio employs in his triangles AEC and CBE. Now, each is seen to be the isosceles triangle on the side of an equilateral pentagon with apex at the center of the circumscribing circle. Referring to Table 2, it will be found that an equilateral pentagon of side 3.7 = 21, will have a chord length of 34, and proportionately will have a radius of 3.6 = 18. This example, is typical of the wit required to find integral values to fit the numerical irrationality of most geometrical objects, especially before the arrival of decimal notation in the seventeenth century, Figure 14.

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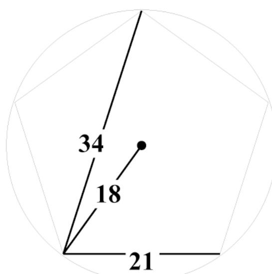


Figure 14. Natural numbers assigned to the radius, chord and side of the equilateral pentagon.

Rachel Fletcher takes drawings of Villa Emo and overlays these with regulating lines. In doing so she follows a time honored analytical methodology. Her overlays show very clearly that the proportional design of the Villa may have been generated by applying the golden ratio consistently throughout. There is no doubt concerning the hypothesis: "Golden Mean proportions appear in the Villa Emo, whose measured drawings suggest that Palladio employed mathematical proportions through a consistent application of geometric techniques"[31], Figure 15.

Essentially, the analysis plays on the well-known property that when either a square is added to the short side of a golden rectangle, or a square is deducted from a golden rectangle, the new issue is itself a golden rectangle[32], Figure 16.†

Figure 15. Below. Villa Emo overlaid with the *section d'or* hypothesis (from Rachel Fletcher, 2000)

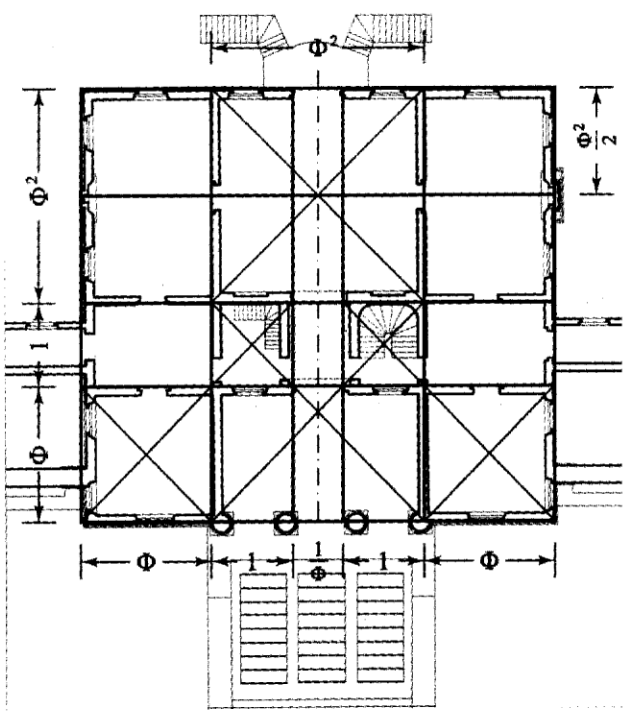
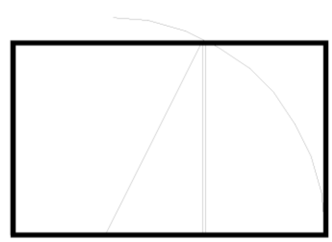


Figure 17. The construction of an extreme and mean rectangle from a square.



† Editors' note: Figure 16 missing from original material.

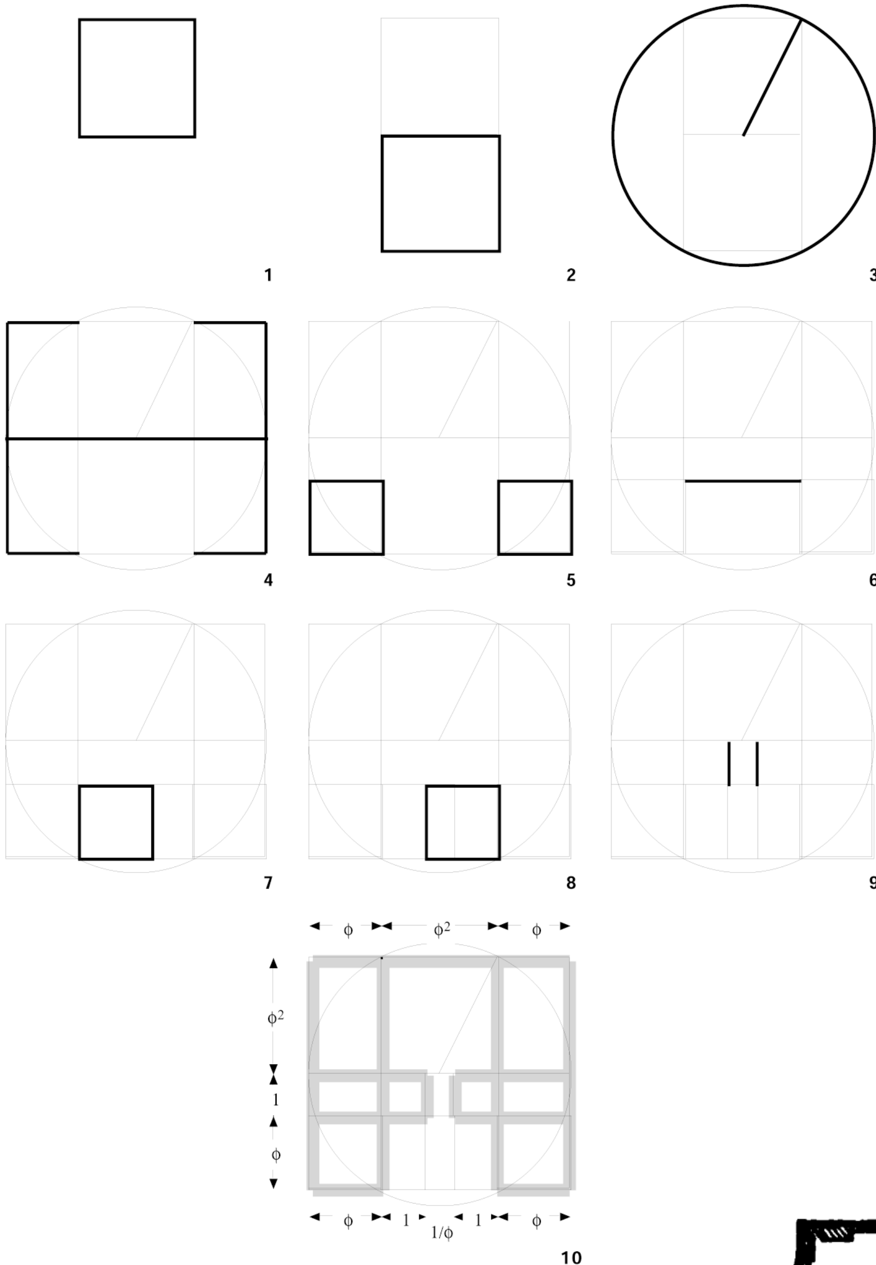
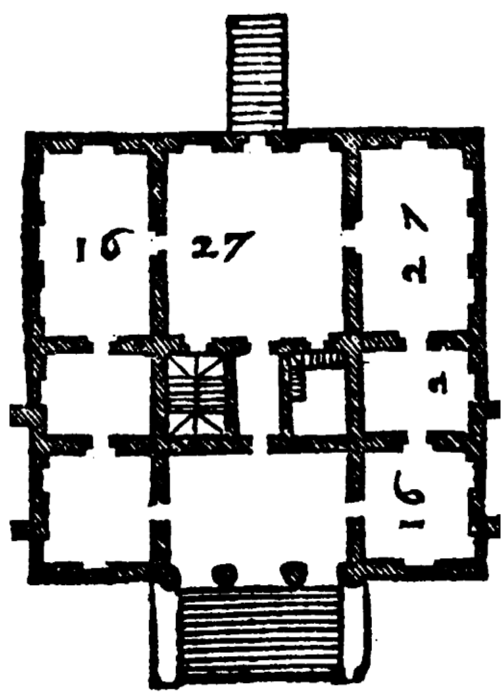


Figure 18. The generation of the extreme and mean ratio (EMR) scheme for Villa Emo.
1. a square;
2. add a square to make a double square;
3. strike a circle to circumscribe the double square;
4. draw the diameter and extend the double square into a rectangle touching the circle;
5. draw two squares to produce two smaller EMR rectangles;
6. complete the EMR rectangle between the two squares;
7. subtract a square from the left side of this rectangle;
8. subtract another square from the right side;
9. complete the small EMR rectangle in the center of the scheme.
10. outline of Villa Emo related to the EMR scheme.

The golden rectangle itself may be generated from the square by striking a circular arc from the center of a side through an opposite corner, Figure 17:

Following this method, the composition of the Villa Emo is generated from an initial square, Figure 18.

Figure 19. Detail of Palladio's woodcut of Villa Emo.



The method replicates the golden proportional scheme with which Rachel Fletcher cloaks Villa Emo. How well does this cloak fit? Visually, it looks fine, but suppose a check is made with the dimensions that Palladio shows on his own woodcut of the project , Figure 19?

A simple model which compares Rachel Fletcher's analysis with Palladio's declared dimensions can be established with two unknowns: x the wall thickness, and y the expected value of ϕ, the *section d'or*, Figure 20.†

If the wall thickness is an unknown x, and an as yet undetermined continuous proportion is assumed for the design $1 : y :: y : y^2$, then the proportion

$$(59 + 4x) : (55 + 4x) :: y\,(y + 2) : (1 + y + y^2)$$

must hold. This requires that the equation

$$(59 + 4x)\,(1+ y + y^2) = y\,(y + 2)\,(55 + 4x)$$

be true. The equation reduces to the parabola

08.10

$$59 + 4x - 51y - 4xy + 4y^2 = 0\ .$$

Set $x = 1$, that is, assume a unit wall thickness. The equation then becomes

$$63 - 55y + 4y^2 = 0.$$

The solutions to this are $y = 1.2611...$ and $12.4889...$

These values do not correspond to the hypothesis that $y = \phi$, the *section d'or*. The first value falls short of the *section d'or* value of 1.618... by almost 12%. The second solution is too way out even to contend.

Suppose that the wall thickness is larger. Set $x = 2$ as a trial. The equation is then

$$67 - 59y + 4y^2 = 0$$

The solutions to this are $y = 1.2398...,\ 13.5102...$.

This is worse than the previous result, and since the function is monotonic, any increase of wall thickness beyond 1 piede will never make things better.

Try the assumption that Palladio has used centerline dimensions. Set $x = 0$.

$$59 - 51y + 4y^2 = 0$$

The solutions to this are $y = 1.28672...,\ 11.4633...$. These are still totally inadequate estimates for ϕ.

What values of x, the wall thickness, will deliver the accepted value of ϕ ? Take the original equation

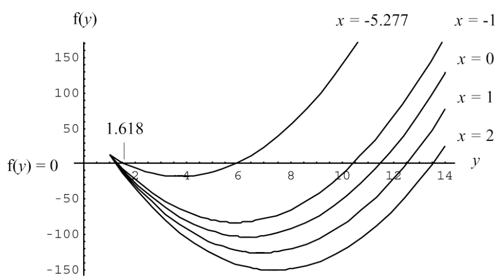
$$59 + 4x - 51y - 4xy + 4y^2 = 0$$

and set $y = = 1.618...$. The equation now reduces to the linear equation

$$-13.046 - 2.489x \approx 0$$

The solution gives $x \approx -5.278$, or a negative wall thickness of over 5 piedes ! The computations may be illustrated graphically, Figure 21.

† Editors' note: Figure 20 missing from original material.



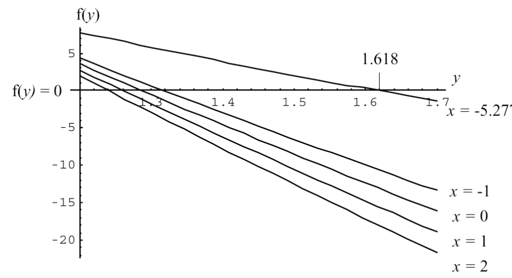


Figure 21. Left. Graph showing the parabolic curves of *f* (*y*) = 0 for five values of x.

Figure 22. Right. Close up view of graph of *f* (*y*) = 0 for values of *y* from 1.2 to 1.7.

For values nearer ϕ, a detail is shown in Figure 22.

These high school computations make it abundantly clear that the golden proportion hypothesis simply does not fit Villa Emo. Drawings deceive, where numbers expose. Alternative explanations for the proportional design of the Villa Emo are given elsewhere.[33]

Villa Emo 'glisters' among Palladio's works., but it is not cloaked in the gold of the *section d'o*r. If the cloak doesn't fit, you must acquit. Palladio is not guilty. But there is plenty of guilt to spread around. The author of the paper which 'saw' the *section d'or* in Villa Emo is an innocent adherent of a morphological church that has flourished since the advent of Zeising's work in 1854[34].

Rachel Fletcher has presented a diligent and exemplary study of its kind. Her misfortune, in casting a cloak of golden proportion over the Villa Emo, is that, unlike the quintessential studies of, say, M Borissavlievitch (1952)[35], or R A Schwaller de Lubicz (1949)[36], her architect, Palladio, has given the actual measurements. In other studies, and in the absence of the architect's specification, the investigator is at liberty to choose where to take measurements and with what precision.: 'With a little precision in taking measurements, it [the *section d'o*r] is easily found'[37]. But such a cloak can never be checked.

What is surprising is that a visually gratifying result is so very wrong when tested by the numbers. It suggests that there is enormous opportunity for visual error in the search for the golden 'whatever', an error that computation exposes ruthlessly. Perhaps, the most alarming consequence of obedience to this morphological faith is that the extraordinary inventiveness, creativity, wit and playfulness of *homo faber* is analyzed into some ideal, universal system, *post facto*[38]. What is this overwhelming desire among some to trade Freedom for Necessity? The obsession with the *section d'or* would be like musicians being fixed solely on the harmony of the common chord, ensuring that everything in their compositions was governed by its limiting proportion.

Palladio had no system of proportion. He was a mannerist. Rules were there to be challenged, to be transformed, to surprise in their unexpected application, or unforeseen consequence. In the process of design, as the dimensions of a work gather around the physical and geometric possibilities and constraints, the designer discerns familiar patterns and potential interpretations. For a humanist during the renaissance, these might include Plato's Timaean myth, the classical orders of number taxonomy, Euclidean geometry, music theory, cosmology, or just plain, practical expediency. It can be assumed that Palladio's work is executed in a polysemic language, foreign to modern eyes: enrichingly ambiguous, despite its enticing presentational lucidity.

Look. Palladio cannot be perceived through over-the-counter prescription glasses.

EXIT D'OR?

08.12

Endnotes

[1]Roger Herz-Fischler, *A Mathematical History of Division in Extreme and Mean Rati*o, Wilfred Laurier University Press, Waterloo,Ontario, 1987.

[2]See Ingrid Rowland, translator and editor, *Vitruvius; Ten Books on Architectur*e, Cambridge University Press, Cambridge, 1999, p47.

[3]Robert Lawlor, *Sacred Geometr*y, Thames and Hudson, London, 1982, p59.

[4]Luca Pacioli, *La Divina Proportion*e, Venice, 1509.

[5]Martin Kemp, *Leonardo: On Paintin*g, Yale University Press, New Haven CT, 1989, p120.

[6]Lionel March, *Architectonics of Humanism: Essays on Number in Architectur*e, Academy Editions, London, 1998. See Essay XII "Inexpressable Proportion'.

[7]Leon Battista Alberti, *Ludi Matematici,* Raffaele Rinaldi, editor, Ugo Guanda Editore, Milan, 1980.

[8]David Fowler, *The Mathematics of Plato's Academy: a New Reconstructio*n, Oxford Sciebce Publications, Oxford, 1987.

[9]James S Ackerman, *Distance Points: Essays in Theory and Renaissance Art and Architectur*e, The MIT Press, Cambridge MA, 1991. See Essay 8 ' "Ars Sine Scientia Nihil Est" Gothic Theory of Architecture and the Cathedral of Milan', pp211-268.

[10]Rudolf Wittkower, *Architectural Principles in the Age of Humanis*m, 50th anniversary edition, Academy Press, London, 1998.

[11]Augusto Marinoni, 'The Writer: Leonardo's Literary Legacy' in Ladislao Reti , editor, *The Unknown Leonard*o, Abradale Press, New York NT, 1974, p70.

[12]J Rykwert, N Leach, R Tavernor, translators and editors, *Leon Battista Alberti: On the Art of Building in Ten Book*s, , The MIT Press, Cambridge MA, 1988, p196.

[13]Rachel Fletcher, 'Golden Proportions in a Great House: Palladio's Villa Emo' *Nexus III: Architectureand Mathematics* ed. Kim Williams, Pacini Editore, Pisa , 2000, pp73-85.

[14]Daniele Barbaro, Venice, Francesco Marcolini, 1556.

[15]Daniele Barbaro, *M Vitrvvii Pollionis De Architectvra Libri Decem Cvm Commentariis* Francesco de Franceschi and Zuane Krugher, Venice, 1567.

[16]Andrea Palladio, *The Four Books on Architectur*e, Robert tavernor and Richard Schofield, The MIT Press, Cambridge MA, 1997, pp62-66.

[17]Alberti gives a written description of the exact method for drawing an equilateral decagon. See note 11 above.

[18]Albrecht Dȯrer, *The Painter's Manua*l, W L Strauss, translator, Abaris Books,New York, 1977, pp144-47.

[19]Sebastiano Serlio, *On Architectur*e,V Hart and P Hicks, translators, Yale University Press, New HavemCT, 1996, p29.

[20]Daniele Barbaro, *La Pratica della Perspettiv*a, Camillo and Rutilo Borominieri, Venice, 1569, p27.

[21]See Roger Herz-Fischler, note 1 above. For a discussion of Kepler, pp159-160.

[22]D'Arcy W Thompson, *On Growth and For*m, Cambridge University Press, Cambridge, (First edition, 1917) Second edition, 1942), 'On Leaf-Arrangement,or Phyllotaxis', pp912-933. Note particularly: "One irrational angle is as good as another: there is no special merit in any of them, not even in the *ratio divin*a", p933.

[23]See Lionel March, *Architectonics of Humanism: Essays on Number in Architectur*e, Academy Editions, London, 1998, Appendix II, Table 2, p278.

[24]Gioseffo Zarlino, *Le Istitioni harmonisch*e, Venice, 1558. 1 : 1, unison; 2 : 1, diapason; 3 : 2, diapente; 4 : 3, diatesseron; 5 : 3, major hexad; 8 : 5, minor hexad. See Claude V Palesca, *Humanism in Italian Musical Though*t, Yale University Press, New Haven, CT, 1985, pp244-254. Palesca, pp235-244, points out that Lodovico Fogliano, *Musica Theori*a, Venice, 1529, had already established the musical provenance of these ratios.

[25]*Ibi*d, p35.

[26]Palladio, Book I, p55. Tavernor and Scholfield, *op ci*t, p60.

[27]See Lionel March, *op ci*t, especially Essay XXVI 'Andrea Palladio'.

[28]Lionel March, *op ci*t, 'Inexpressible Proportion', pp65-69.

[29]Palladio, Book I, *op ci*t, p57.

[30]Thomas L Heath, *op ci*t, pp455-457.

[31]Rachel Fletcher, *op ci*t, p78.

[32]For a generalization of this, see Lionel March, 'Architectonics of proportion: a shape grammatical depiction of classical theory', and 'Architectonics of proportion: historical and mathematical grounds', *Planning and Desig*n, 26.1, pp91-100, and 26.3, pp447-454.

[33]Lionel March 'Palladio'sVilla Emo: the golden mean hypothesis rebutted',submitted to NEXUS, 2001.

[34]A Zeising, *Neue Lehre von den Proportionen des menschlichen Kφrper*s, Leipzig, 1854.

[35]M Borissavlievitch, *The Golden Number: and the Scientific Aesthetics of Architectur*e, Alec Tiranti, London, 1958.

[36]R A Schwaller de Lubicz, *The Temple in Man: Sacred Architecture and the Perfect Ma*n, trans R and D Lawlor, Inner Traditions International, Rochester VT, 1977.

[37]*Ibi*d, p66.

[38]D'Arcy W Thompson, *op ci*t, "... all such speculations as these hark back to a school of mystical idealism", p933.