

Configuration and Centrality

Some evidence from two Italian case studies

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Abstract

The research this paper is related with concerns the analysis of the correlation of the distribution of the configurational indexes versus the presence of the activities actually located in two Italian urban cases. The research is aimed at verifying the reliability of the methods of configurational analysis as a predictive tool regarding the availability of each part of the urban settlement to recipe activities and let them flourish and to define, on the other hand, the actual range of their reliability. In other words, that's to say that the obtained results allow to assume a new approach to the theme of urban centrality, appraised in term of attractiveness: setting aside the actual presence and the position of the located activities, centrality, on the basis of the results so far, may be regarded as a spatial process, the levels of attractiveness of each part of a settlement being directly related with the development of the grid, and appraised by means the analysis of its configuration. Moreover, on the basis of those results, the location of monopolistic activities stands out as a strategic variable in town planning and management.

Introduction

The notion of centrality is one of the fundamental crux where town planning splits off from urban theory and modelling. On the one hand, in fact, the managing of urban centrality is repeatedly stated as a fundamental purpose of many recent town plans and projects: plans aimed at decentralising blocked urban cores, plans aimed at enhancing the centrality level of peripheral areas, plans aimed at revitalising ancient urban cores which have recently lost their previous centrality for the benefit of new development areas, got more accessible and attractive.

On the other hand, it can be noticed the absence of reliable tools suitable for managing urban centrality, which often makes the mentioned plan purposes actually inefficiency, generic or just a merely abstract statement. This lack is likely to be due to the vagueness and to the ambiguity the notion itself of centrality, though certainly one of the most frequent assumption of urban theorists and operators, reveals, as it's generally regarded in urban planning practice and theory, where the term appears mentioned from various points of view and with several different meanings. Urban centrality can be appraised from an historicist point of view, merely (or mostly) considering a central urban area as the place where the historic memory of the community appears mainly materialised, stratified and conserved. Again, the same notion is often assumed from a prevalent architectural point of view, which refers the centrality of an urban place to the presence, the physical shape and the morphology of blocks

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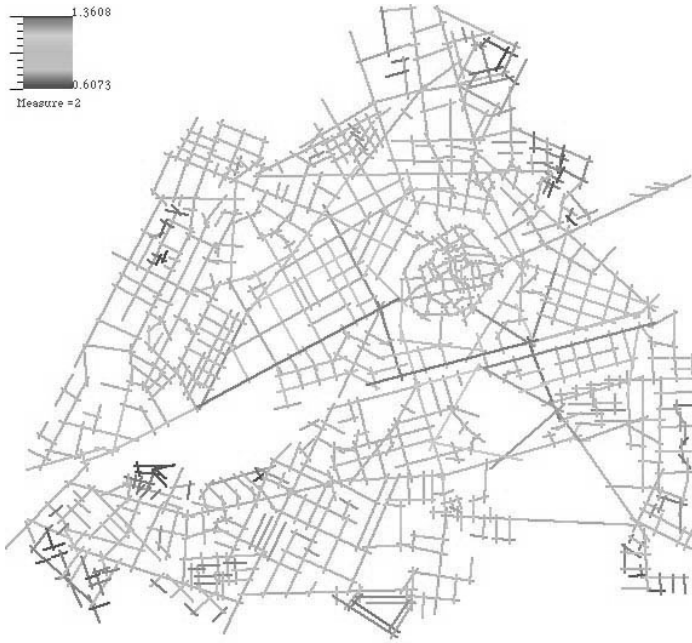
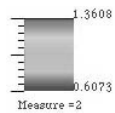


Fig. 1 – Distribution of global (radius = n) integration value in the grid of Grosseto

and buildings. Urban modelling (and in particular the studies coming from classic location theory) generally approaches the theme of urban centrality from a functional point of view, regarding its notion in terms of attractiveness, and referring it to the presence, the density and the typology of the located activities (Losch, 1952; Isard, 1956; Alonso, 1964; Herbert and Stevens, 1960): roughly speaking, an urban central place is said to be a place where activities aim at getting located, and the struggle for such a location can be seen as the hierarchical principle of the internal geography of towns. Clearly, that appears quite a tautological approach (that's to say '*a central place is a central place*') and above all it doesn't show the reasons which primarily determine this attractiveness.

Yet, all these kinds of approach to centrality (even the functionalist one) can't really account for the actual causes of the primary location of the activities into the central places, nor for the causes that frequently make those activities to move from a place to another, searching for a more favourable position.

The recent phenomenon of the shifting of centrality from the ancient urban cores towards new development areas clearly highlights this inadequacy: those areas are absolutely devoid of historic significance, are generally deprived of any architectural or morphological

pregnancy, in the beginning are certainly lacking in functional importance; yet, the same areas do attract activities, which often leave the ancient urban nuclei in order to find a more favourable location in a more 'central' (that's the word) place. What do attract them? What do they miss in the historic centres? Or, what's probably better to say, which is the quality that once made the ancient cores so attractive (that's to say so central), and that they have subsequently lost for the benefit of new development areas? And again, on the contrary, what do make most of our peripheral areas so segregated and marginal and what's the way to overcome this marginality? Understanding it means understanding what essentially centrality is, and, above all, it involves the possibility of controlling (either boosting or opposing) the dynamics that makes it to shift from an urban area to another, to appreciate or, on the contrary, to depreciate an urban area or another.

That's the reason why we're now looking for a different point of view, a configurational point of view: instead of being regarded as a historic heritage, a morphological status or a functional condition, urban centrality is then to be appraised as a spatial process (Hillier, 1999), directly related with the continuous development of the urban grid. As soon as the grid changes in consequence of any urban transformation, then even the distribution of the attractiveness levels in the grid itself changes, modifying the position of the central areas. Each urban place, hence appears characterised by an own centrality level, that has to be considered with reference to all the other parts of the settlement, regarded as a system. Here configurational analysis comes into play.

With reference to the problem of the prediction of pedestrian flows in towns, in our researches so far (Cutini, 1999a) that ideally carried on researches on movement and configuration (Hillier et al, 1993; Hillier, 1996a; Hillier 1996b), we proved the existence of a significant correlation between the configurational indexes and the pedestrian movement rates. The correspondent relation resulted typically exponential, and we expounded it as the outcome of the multiplier effect caused by the attracted activities working, on their turn, as movement attractors.

The index that the movement rates resulted most correlated with was the local (radius 3) integration value; besides, we had to notice (Cutini, 1999b) that this correlation, though far stronger than we could expect, got dramatically poorer when tested all over the urban grid. This effect was supposed to be caused by the presence of strong global attractors which had been located in the grid without any reference to its configuration.

Now, we bring land use into action, aiming at verifying the existence and the strength of a correlation between the presence of activities and configuration. Only if activities were actually proved to be strongly related with configurational indexes, we could assert their presence and respective location to be influenced by the grid; such an influence would assert the grid configuration as the primary cause of the production of inequalities with regards to attractiveness. In other words, if we prove the distribution of the levels of centrality to be function of the configuration of the grid, we may assert the possibility of predicting the effects of any planned transformation of the urban grid on that distribution. And, even more, we may assert our capability of evaluating, by means of configurational analysis, the compatibility of the planned destinations with the resulting attractiveness level in their respective location area. In short, we'd have at our disposal a tool suitable for verifying the change of the level of centrality in each part of an urban settlement as a consequence of any development town plan or project.

The method sketched above has been applied to our selected case studies: two Tuscan towns, Grosseto and Orbetello, the same cases we previously studied with reference to the prediction of pedestrian movement.

Case studies

Our research has been applied to the case studies previously selected, namely the Tuscan towns of Grosseto and Orbetello.

The axial maps of the two urban grids had already been constructed, consisting of about 800 lines in the case of Grosseto (a bigger settlement, with about 70,000 inhabitants) and of around 150 lines at Orbetello (15,000 inhabitants). The subsequent processing of the axial maps by means of Axman Software (Cutini, 1999a; Cutini, 1999b) provided for each single line of the two cases a complete set of the values of the configurational indexes. Axman provides also a chromatic representation of the trend of every configurational indexes in the

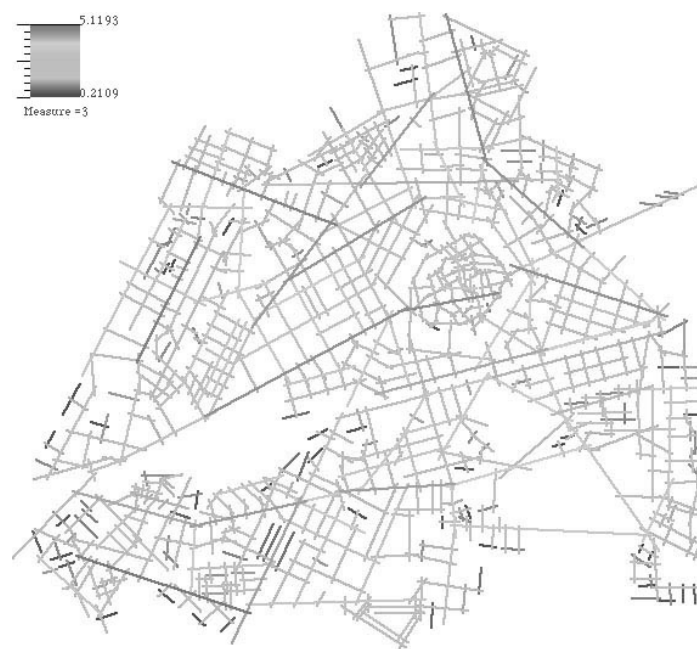
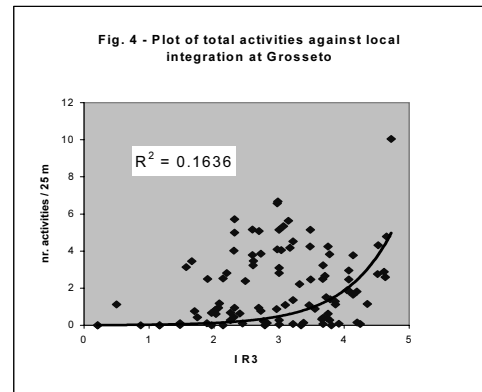


Fig. 2 – Distribution of local (radius = 3) integration value in the grid of Grosseto



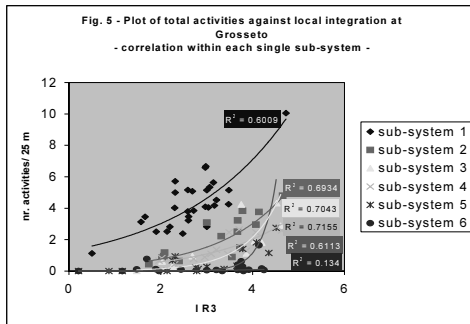
lines of the grid (see figures 1 and 2); in particular, the trend of the integration (both global and local) is described with warm colours (up to orange and red) for the most integrated lines and cold colours (starting from violet and blue) for the ones with poorest integration values.

We have then pointed out the presence of the activities in the two settlements. Starting from Grosseto, we've selected 100 lines, well distributed all over the grid, from the centre up to its edge. The observed activities have been classified, with respect to their own characteristics, in six fundamental categories: primary commercial activities (food-shops, bar, tobacco-nists, chemist-shops, etc.), secondary commercial activities (clothes, books, computer-shops, restaurants, etc.) financial activities (banks, insurance companies, etc.), professional and crafts-man activities, political and administrative activities as public offices, cultural and religious activities (schools, libraries, museums and exhibitions, churches, etc.). Moreover, we've made a further distinction, distinguishing the monopolistic activities from those operating in a free market.

Since the topological approach involves each line of the urban grid to be simply appraised as a single path unit, it was necessary to standardise the consistency of the observed activities assuming as its measure the number of the activities per 25 meters, so that the actual length of each line has been left out of any consideration.

Hence, we started focusing on the correlation between the configurational indexes (resulting from the automatic processing of the axial map) and the present activities (resulting from direct observation).

The analysis by linear and non-linear regression of the correlation of the total number of the observed activities against the global integration (radius n) index all over the grid provided disappointing results, with a dramatically poor $R^2 = 0,051$. Still poor results were obtained considering the activities disaggregated according to the various categories: none of them resulted significantly correlated with the same configurational value. We then took into account the local (radius 3) integration value, instead of the global one, and here things went better: we had a $R^2 = 0,164$ for all the activities taken together (see Fig. 4), and similar results ($R^{21} = 0,145$, $R^{22} = 0,178$, $R^{23} = 0,134$, $R^{24} = 0,144$, $R^{25} = 0,0016$, $R^{26} = 0,0005$) for their several categories one by one. It's easy to notice that free market activities appear more correlated with the integration values than the monopolistic ones. Nevertheless, though higher than the previous results, still all those coefficients remain far lower than an acceptable significance standard.



	regression curve $y = KexpWx$	whole grid	sub-system 1	sub-system 2	sub-system 3	sub-system 4	sub-system 5	sub-system 6	sub-system 1 - correlation per groups - (7 groups of 8 lines)
all kinds of activities	K	0.0085	1.286	0.196	0.0103	0.1704	4 E-06	0.0011	2.0635
	W	0.0013	0.0004	0.0007	0.0013	0.0006	0.0031	0.0011	1.5637
	r^2	0.164	0.601	0.693	0.714	0.716	0.611	0.134	0.946
only free-market activities	K	-	1.0771	0.1291	0.0088	0.0965	4 E-06	9 E-07	1.7706
	W	-	0.0004	0.0008	0.0013	0.0008	0.0031	0.0013	1.5311
	r^2	-	0.718	0.757	0.729	0.717	0.613	0.116	0.966

Bearing in mind the results of our previous researches (Cutini, 1999a), our further attempt was then the disaggregation of the analysed system from the whole urban grid to several limited lines sub-systems, which have been defined on the basis of the same available configurational indexes: the criterion for the definition has been the presence within each sub-system of a strong local integrator and of all the observed lines lying within a 3 steps topological radius from them. We thus obtained 6 sub-systems, respectively composed of 26, 15, 10, 13, 16 and 20 lines (see Fig. 3).

The analysis of the correlation of the observed activities with the integration (local integration, of course) value finally provided excellent results, which actually can be regarded as strongly significant (see Fig. 5): $R^2_1 = 0.601$, $R^2_2 = 0.693$, $R^2_3 = 0.704$, $R^2_4 = 0.716$, $R^2_5 = 0.611$ and $R^2_6 = 0.134$.

We then focused on the relation between the local integration index and the only non-monopolistic activities, and the results confirmed the correctness of our previous conclusions, providing the following correlation coefficients with reference to the several selected sub-systems: $R^2_1 = 0.718$, $R^2_2 = 0.757$, $R^2_3 = 0.729$, $R^2_4 = 0.717$, $R^2_5 = 0.613$ and $R^2_6 = 0.116$. All these results are summarised in the table 1.

Two aspects appear worth highlighting:

1. Though the differences are somewhere minimal, nevertheless we can easily notice that in all cases the position of the free market activities reproduces the distribution of configurational values more narrowly than the monopolistic activities: the presence of activities operating in a monopolistic market, on the contrary, appear likely to deform the correspondence between configuration and land use within each sub-system. That's particularly evident in the first two sub-systems, which respectively correspond to the historic centre and to the railway station area, where the presence of monopolistic activities (public administration, public offices, utilities, cultural, political and religious activities) is far larger than in the peripheral areas.

2. Moreover, we can notice that only in one case (the sub-system nr. 6) the examined correlation is steadily poor, but this discordance even confirmed the effectiveness of the results themselves: in fact, while the other 5 sub-systems correspond to older, typically multifunctional areas, which have been growing up spontaneously, the sub-system nr. 6 actually corresponds to a recent development area, urbanised since the 70s by means of a zoning plan characterised by a strong functional division between (few) commercial districts and (many)

Table 1 – Main results of the correlation analysis (activities versus local integration) in the urban case of Grosseto

residential streets. Shops and offices just were not allowed to take their place in those street, setting aside the possibility of a spontaneous choice of the most favourable location of each activity.

Unfortunately, those results, though much higher than the significance standard, were depreciated by the actual small consistency of the data (from 10 to 26). Aiming at verifying the meaning of such results on a more consistent (and therefore significant) data base, we've then changed our work scale, focusing on the correlation analysis between land use and configuration in all the lines of a single urban sub-area. As our new case study, we've chosen the sub-system gravitating towards the local integrator actually named Corso Carducci, the main street of the ancient town; that area roughly corresponds to the historic centre of Grosseto, still encircled within its ancient townwalls. This choice involved to take into account 56 lines, and of course to carry out the direct observation of the presence and consistency of all the activities (disaggregated as in the studies so far) in each of them.

Aiming at cutting down the effects of single lines' land use (that are likely to be affected by local and incidental elements), we divided the observed 56 lines in 7 groups of 8 lines per group, tidied up according to the local integration value (starting from the strong integrators group up to the group containing the most segregated lines). Finally, we checked the correlation of the correspondent 7 integration mean values against the respective activities total consistency. In this case the results were definitely excellent: the relation was proved to be exponential, with a correlation coefficient $R^2 = 0,946$. And, as usual, the analysis regarding only the non-monopolistic activities provided a even higher result (see Fig. 6 and Fig. 7): $R^2 = 0,966$, which obviously can be regarded as almost perfect.

We then applied all the procedure carried out so far to our second case study, regarding the town of Orbetello. On the one hand, we constructed the axial map correspondent to its urban grid and we processed it by means of the same Axman Software previously used; on the basis of the resulting configurational indexes (see Fig. 8 ad Fig. 9) we then defined its two resulting sub-systems, the sub-system roughly correspondent to the town's historical centre and the one coinciding with the more recent eastwards growth area.

On the other hand, we carried out the direct observation of the present activities along the 69 lines that belong to the sub-system 1 and the 40 lines lying in the sub-system 2, disaggregated as previously specified.

The correlation (activities versus integration) was analysed with reference both to the whole grid and to the two selected sub-systems. And, again, the results were better in this last case. The correlation of local integration index versus the observed presence of activities in the lines of the historic centre, though not really disappointing, wasn't actually so strong (revealing an exponential curve characterised by $R^2 = 0.600$ in the case of the total number of activities and 0.691 taking into account only the free-market ones (see Fig. 10 and Fig. 11)).

In the sub-system 2, the same analysis provided better results, with an exponential correlation and a coefficient $R^2 = 0.749$ in the case of free market activities (see Fig. 13).

Again, also in this second case the correlation values can easily be enhanced clearing off the effect of the fortuitous presence (or absence) of activities in many single lines, which are likely to be affected by contingent or local causes. As above, we therefore tried to clear out these contingent (and not generalizable) elements by means of the disaggregation of all the lines in 7 groups of 10 lines and studying the correlation of the mean local integration value of each group with the respective group total number of activities (per length unit, of course).

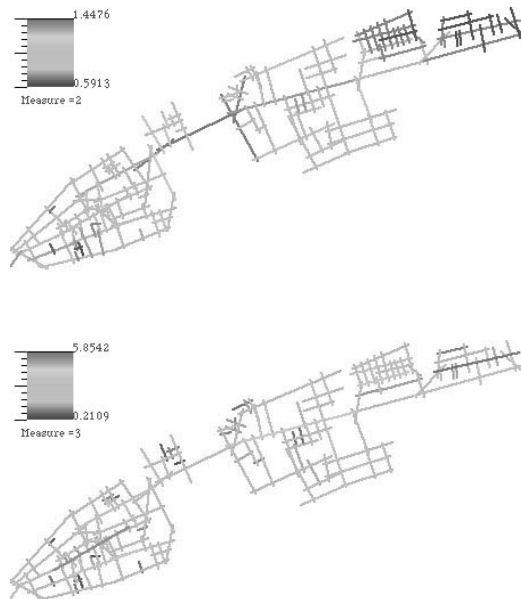
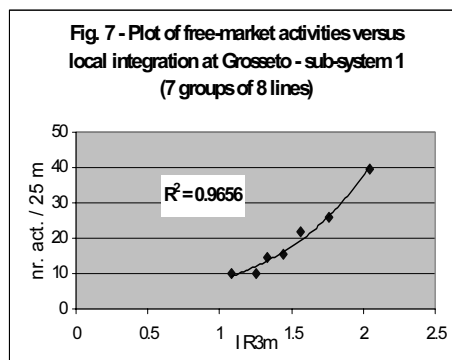
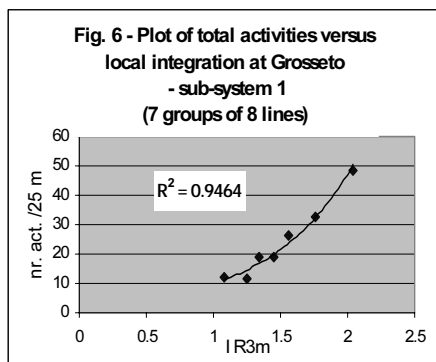
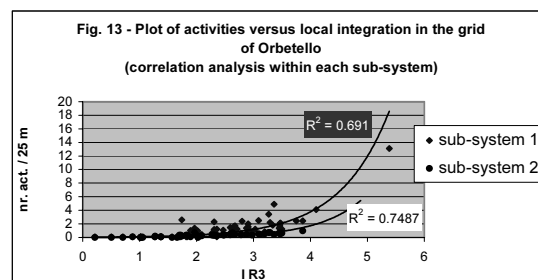
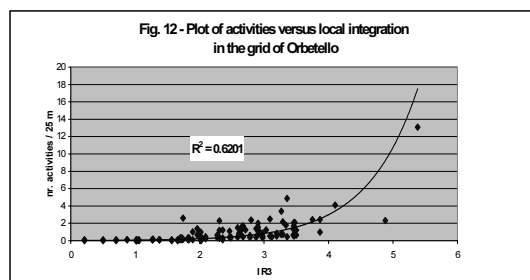
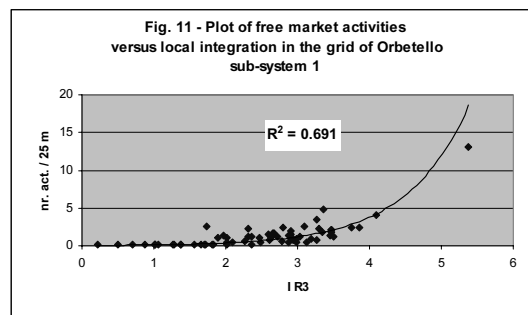
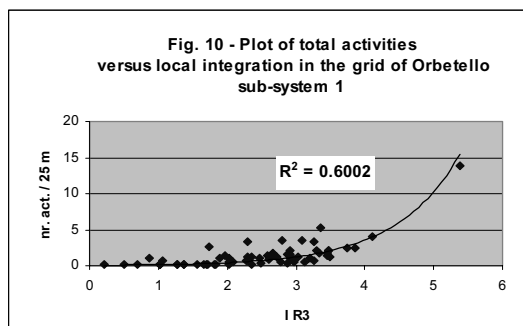
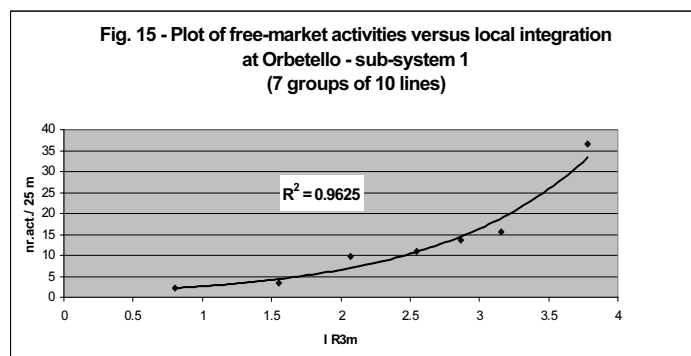
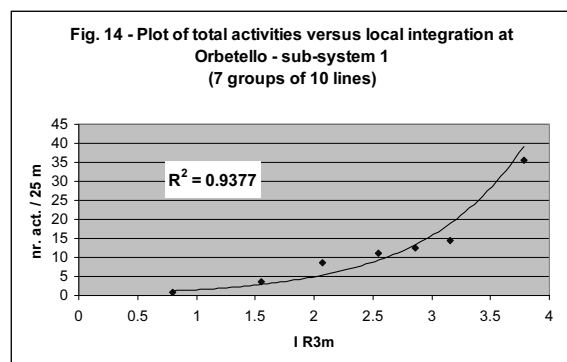


Fig. 8- Distribution of global (radius = n) integration value in the grid of Orbetello

Fig. 9 - Distribution of local (radius = n) integration value in the grid of Orbetello

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In this case we got excellent results (see Fig. 14 and Fig. 15), as the exponential relation was proved by a coefficient $R^2 = 0.938$ with all the observed activities and even $R^2 = 0.963$ if excluding the monopolistic ones.

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Conclusions

The results of the researches so far can then be briefly summarised as follows:

- a correlation between the presence of activities and the grid configuration does exist, and is stronger than we expected;
- the form of this correlation is clearly exponential;
- in a local scale, the configurational index activities are best correlated with is local integration;
- the correlation, though very strong within small sub-systems of the grid (which correspond to areas defined by the lines gravitating towards a strong local integrator), weakens when analysed all over the settlement;
- the distribution of global integration values influences the correlation of local integration versus the presence of activities in the several sub-systems;
- the correlation, though very strong in areas which have been growing up spontaneously, weakens in areas resulting from development zoning plans;
- the correlation of configurational indexes is stronger with the presence of free-market activities rather than with monopolistic ones.

Such results can be easily interpreted.

The exponential form of the correlation seems to account for the multiplier effect activities work out: they are attracted by the grid configuration towards the most favourable position they can achieve, but their location itself, on its turn, attracts other activities.

Centrality, defined in terms of attractiveness, is hence proved to be appraised as a function of configuration. Each part of the settlement, in fact, seems characterised by an own centrality level, that seems to depend on a configurational parameter, the global integration value of the strong local integrator of the sub-system, and on several other (non-configurational) elements, which can be summarized as follows:

- the presence of *strong global attractors*, that is prominent activities characterised by a market radius extended all over the grid;
- the demographic density of the area;
- the presence of specific *utilities* and *amenities*; among those amenities, it's worth inserting also the historic pregnancy and the architectural value that characterise the observed sub-system. As we sketched above, those element

are sometimes regarded as the main indicators of urban centrality; in our research, we assume them to concur (with the other mentioned elements) in raising the attractiveness level of their respective urban sub-system.

All these elements seem to work together to determine the coefficient K and W of the expression

$$y = K \exp Wx$$

(where x is the local integration value and y the number of activities of each line), that has been proved to reproduce within limited urban sub-areas the relation connecting configuration and land use. That's to say that all those elements concur in determining the way configuration influences land use in each sub-system.

For instance, in the case of Grosseto we resulting values of those coefficients are included in the Table 1.

That's to say that the plot of activities versus integration, appearing just as a wide cloud of scattered dots (see Fig. 4 and Fig. 12), otherwise can be easily (and usefully) seen as the aggregation of several (six in the case of Grosseto, just two in the grid of Orbetello) groups of points, narrowly correlated around their respective sub-system's exponential curve (see Fig. 5 and Fig. 13).

In other words, activities and integration seem to be strongly correlated all over the grid by means of the expression seen before. Nevertheless, the coefficients K and W which specify that correlation change while moving on the grid, and their value can just be defined within each of the sub-systems, on the basis of both configurational and gravitational elements.

This kind of description, moreover, allows to define and to clearly distinguish the actual level of centrality of the several sub-systems of an urban settlement, which is represented by their respective regression curve: the higher the position of the curve appears in the diagram, the stronger the attractiveness of the correspondent sub-system will result. In the analysed case of Grosseto, for instance, it's easy to notice the highest attractiveness level of the historic centre, substantially coinciding with the sub-system 1, with respect to all the other parts of the settlement, and especially to the most peripheral ones, lying at the bottom of the diagram.

On the other hand, each single curve reproduces the distribution of centrality within the correspondent sub-system, according to a clear exponential trend, described by the expression seen before.

Working on *small* parts of an urban settlement, we can thus argue the possibility of predicting the distribution of attractiveness in all their respective lines, taking into account only the grid configuration and setting aside the presence and the actual position of the located activities; or, in other words, we can assert, in the same hypothesis, the reliability of configurational analysis as a predictive tool regarding the availability of those lines to receive activities and let them flourish.

Of course, this dynamics refers only to activities which are free to move and to choose their own position on the grid, while it weakens if they are compelled to follow the location choice of the planner. In this last case, which is generally likely to run out on a short term, a located activity can do nothing but trying to succeed in flourishing, in growing (and often just

in surviving) despite its specific location, if that were not favourable enough to let it cope with the market competition. On a long term, too rigid and unfavourable regulations get generally fitted to the activities' needs.

Again, this dynamics refers in particular to the activities which have to match with the market competition, while monopolistic activities can choose their location on the grid setting aside its configuration. Monopolistic activities can therefore be assumed as non-configurational activities, and their actual distribution, hence, is likely to deform a narrow correspondence between configuration and land use.

Such an inference is going to be strongly confirmed by the results of a research on a further case study, concerning the town of Pisa. The results of this research, which at the moment is not yet concluded (it will be published in the near future), can anyhow be here roughly outlined. The configurational analysis of the urban grid clearly shows the integration core corresponding to the lines around *Corso Italia* (the street connecting the central bridge of Pisa, namely *Ponte di Mezzo*, with the central railway station), in the southern part of the town, on the left side of the river Arno (see Fig. 16). This certainly corresponds to the actual distribution of activities, since Corso Italia is by far the most attractive and crowded shopping street of the town; and even in a smaller scale, we can verify the presence of activities along the lines surrounding Corso Italia to be correlated (still exponentially!) with the distribution of local integration values. Nevertheless, we must discuss the question of the north-western part of Pisa, (in the opposite side of the Arno), which is well known to be an attracting urban area towards activities despite the not-so-high resulting values of global integration. A more careful analysis will show this attractiveness being determined by the presence of some monopolistic activities which can certainly be regarded as strong global attractors: the University of Pisa (40.000 students on a population of 100.000 inhabitants), the Santa Chiara Hospital (one of the most important Tuscan hospitals) and, by far the most prominent, the monumental area of Piazza dei Miracoli, with the famous leaning Tower. All these activities concur in raising the centrality level of a part of Pisa which otherwise the actual grid configuration would probably make poorly attractive.

Monopolistic activities, therefore, are confirmed in their role of non-attracted attractors: they can get located without any reference to the configuration and, on their turn, they attract other (free-market) activities, which aim at taking benefit of the movement they produce.

Yet, for what specifically concerns monopolistic activities, two aspects appear worth highlighting:

1. Actually, a non-configurational location of a monopolistic activity anyhow involves an extra-cost, that obviously won't be suffered by the activity itself as a smaller profit, but by its users, in terms of increasing transport costs; in other words, they won't be damaged by an unfavourable position on the grid, as people using them will probably be.

2. As not constrained by the grid configuration, the matter of the location of monopolistic activities can be regarded as a strategic variable in town development planning and management: their location, in fact, may be preciously utilised to enhance the attractiveness of urban areas which result lacking in configurational centrality. In order to describe this effect, we can say that the presence of monopolistic activities actually modifies the attractiveness-integration curve: more in detail, their presence increases the K coefficient value, raising the local curve in the diagram and therefore enhancing the centrality level of the respective sub-

system. On this basis, monopolistic activities can be used to change the position in the diagram of the attractiveness curve of a local system in order to modify (either increasing or reducing) its centrality level.

On this regards, two cases are worth highlighting as typical and frequent:

1. Urban decentralisation plans aimed at relieving blocked central areas: in this case, the shifting of monopolistic activities can well support the flourishing of free-market activities in segregated areas.
2. The falling-off of attractiveness of the ancient urban cores and the shifting of centrality towards new development areas: in this case, the configurational analysis will highlight the loss of centrality and its actual causes. And, here, the preservation of the monopolistic activities can allow the survival of the other activities in the old towns and the survival and the preservation of the ancient centres themselves.

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Fig. 16 – Distribution of global (radius = n) integration value in the grid of Pisa