Angular Analysis

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Abstract
In this paper we present angular analysis, a spatial analysis technique based on space syntax methods. Angular analysis can either be applied as an extension of axial analysis or visibility graph analysis. We make a case for the introduction of angular analysis to improve on people movement forecasting as provided by space syntax, through appealing to a logical argument about why space syntax might forecast people movement in the first place. We then go on to show two case studies where we have applied the technique. We show, at least within a building context, that angular analysis can improve on people movement forecasting. Finally, we present an argument for why space syntax has performed as well as it has to date, and suggest that using angular analysis, we may be able to improve our understanding of spatial systems.

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Angular Analysis,
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1 Introduction

The purpose of this paper is to introduce angular analysis to the space syntax community. Angular analysis originates from a technical paper (Turner, 2000) where it is presented in a form which is essentially an extension of visibility graph analysis (Turner et al., 2001), although recently it has been implemented as an extension of axial analysis (Dalton, 2000, 2001). Here the technicalities will be preceded by a case for why applying angular analysis might be a reasonable idea for space syntax in the first place, with reference primarily to axial line integration (Hillier and Hanson, 1984). After making this proposal, section 3 discusses the implementation of angular analysis for both axial line graphs and visibility graphs, and discusses the relationship of axial analysis to visibility graph analysis in this context. Section 4 demonstrates the application of the method to two case studies - one building example where applying angular analysis shows improved people movement forecasting, and one qualitative urban example which demonstrates that it may also be fruitfully applied in traffic and pedestrian movement forecasting. The paper is concluded with a discussion of what we might learn about the integration measure in the light of these findings in section 5.

Before starting on a case for angular analysis, let us provide a brief introduction to what angular analysis actually is. In essence, angular analysis uses a weighted graph to calculate space syntactic metrics rather than the non-weighted standard measures. It is easiest to think of it in terms of an axial map, as proposed by Dalton: when calculating the path length from A to B, as one would to calculate integration for example, rather than count the number of edges (that is, connections) between those locations, calculate instead the weighted sum of the edges, where each edge is weighted by the angle of connection. As an example, the intersec-

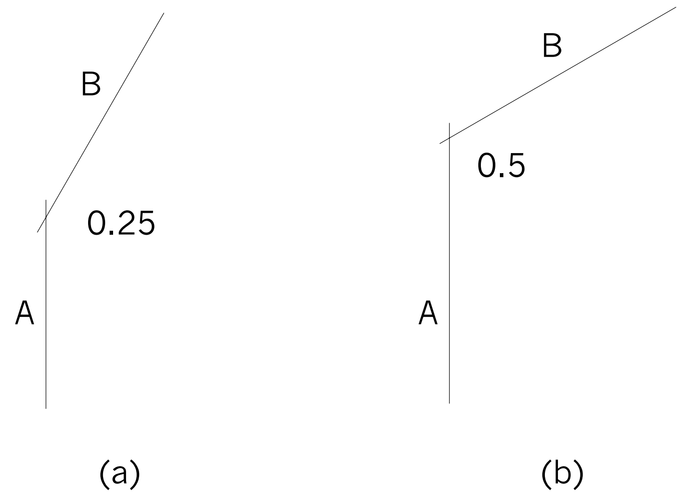


Figure 1: (a) A connection joining two axial lines at 30 degrees to each other is weighted by 0.25 (b) A connection at 60 degrees is weighted by 0.5.

tion of two axial lines at angle of incidence 30 degrees might have a weight of 0.25 (figure 1a), while the intersection of two axial lines at 60 degrees might have a weight of 0.5 (figure 1b). The path length from A to B in figure 1a is therefore 0.25 while in figure 1b it is 0.5. We shall return to a full description of the methodology in section 3, but first let us turn to a case for why we should want to perform such an analysis in the first place.

2 The case for angular analysis

A large body of research has grown up around the integration measure in space syntax. It is accepted as the most consistent measure within the field for correlation with pedestrian movement (Hillier, 1999), at least when calculated locally. The classic study is described by Hillier et al. (1993) where they find that the natural logarithm of movement correlates with the integration value, with a correlation coefficient of 0.547 for an axial map for large area around Kings Cross, among other case studies. Integration has since been shown to correlate well with movement in many other studies. But could we improve on it? As Hillier (1999) looks at axial maps he discovers an interesting phenomenon. The maps split into near right-angle connections - 'within 15 degrees of 90 degrees'. He suggests a consistent constructive process is at work. Other researchers have also found interesting and similar properties of angles in systems: Sadalla and Montello (1989) show that subjects' memory of turns is better for right-angles than other angles, and if in doubt, angles are rounded to the nearest right angle. Of course, this is not a new idea, the suggestion that subjects linearize memory of routes stretches as far back as Lynch (1960). There is more: Montello (1991) shows that people are less able to place themselves when the grid is deformed than when it is not; Conroy (2001); Conroy Dalton (2001) shows that even when taking a route, people linearize it, taking shallower turns towards their goal.

So, what does this have to do with integration? Well the point is that axial integration is a measure of the depth in terms of the number of turns from A to B, biasing all turns equally, be they 1 degree or 90 degrees, while the evidence we have presented shows that some turns are more important than others to humans. A slight shift of 15 degrees is not considered a turn (neither in remembering one's route, nor in locating oneself, nor routing oneself), while anything like 90 degrees is considered to be a right angle. In fact, Hillier is quite clear - there appear to be just three types of human turn: no turn, fork, or right angle. In summary, integration correlates well with people movement, apparently by considering binary turns, while people apparently move by considering some more subtle approach to turns.

To continue we must assume that integration is correlating with movement due to the way that people move around the system. Note that this is in no way proven - our evidence (Hillier et al., 1993; Hillier, 1996; Read, 1999, etc.) merely points out that there is some correlation between movement and integration, or mean depth, not that the mean depth is a causal factor for movement. However, for now, let us assume that axial mean depth of the system is responsible for how far people penetrate the system from any point (note that other researchers also talk in these terms, see in particular Choi (1999)'s discussion of possibility in systems). We will return to discuss our assumptions in the light of our findings in section 5. Now, given that movement is found to be logarithmic with integration, or, within a single system, logarithmic with mean depth, we can construct a simple argument to show that there is a plausible causal link between the two as follows. If N people start out from location A to any other location in the system, how thinly will they spread out among the system? Well if the graph has some mean depth from A, L_A, then on average any path taken from A will end up of length L_A. If we consider the j graph from the point of view from A, we will expect it to have a width at any depth along it according to a splitting factor. In a random system, the number of concurrent paths at any depth within the j graph is described by a Poisson distribution, since splitting is a discrete random event of some probability from the point of view of the j graph from a node (for systems with large depths we should note that this approximates a binomial distribution). For our purposes, all we need to say is that at the mean depth at L_A the graph will be widening approximately exponentially with a near constant splitting factor, ξ . Equation 1 shows this approximation of the Poisson distribution giving the width ν of the graph at depth d:

$$\nu = \xi^d \tag{1}$$

Now let us look at how that will affect a group of N people setting off from location A. Ifthey take any path to depth L_A then they will be split into ν groups, as shown in equation2.

$$N = n\nu = n\xi^{L_A} \tag{2}$$

Now we can rearrange equation by logging both sides to arrive at equation 3:

$$\ln N = \ln n\xi^{L_A} = \ln n + L_A \ln \xi \qquad \Rightarrow \qquad \ln n = \ln N - L_A \ln \xi \tag{3}$$

In words, the people have thinned exponentially according to the mean depth, i.e., the logarithm of the number of people is proportional to a constant (the number of people total in the system) minus the mean depth. Inverting the mean depth value as we do for integration tells us the logarithm of the number of people is proportional to some constant plus the integration, which is what we observe in real systems. However, despite the striking similarity of this equation with observation, there are several caveats - the argument is assumption laden - we must at least note that the equation given is for thinning of people from a location rather than the number of people arriving at a location. This is not to say that this is not a plausible mechanism though: it seems reasonable that people arriving at a destination will coalesce in exactly the same way as they thin from a location.

Returning to our thread, the argument just given suggests that if people flow through a system according to the number of binary turns in the system, then they thin exponentially. But what if people do not look at a route as ‘any deviation at all equals turn’, but, for example, as ‘over a certain threshold then it is a turn’ (as the evidence suggests). Enter angular analysis. We can formulate the same argument, but, as a second approximation to real movement decisions, simply replace the standard j graph with an angular j graph, where every angle turned through weights the edges of the graph according to the amount of deviation from straight. Thus, the probability of taking a turn on when traversing this j graph is based on

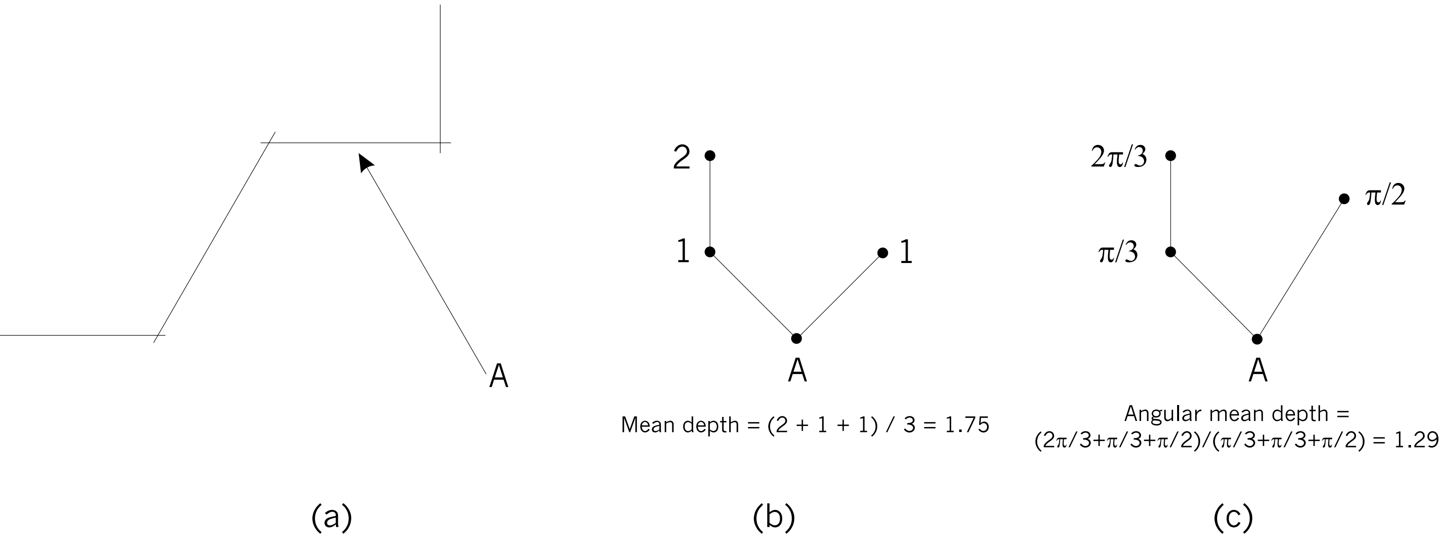


Figure 2: (a) A very simple axial system. (b) Standard mean depth calculated from line A. (c) Angular mean depth calculated from line A.

what angle the new path splits off from the j graph. This suddenly gets much closer to the human view of the j graph - although note, our analysis is linearly weighted with increase in angle while the evidence presented suggests turns are categorised by humans as turn or no turn according to some cut-off angle. Now if humans are allowed to flow through this j graph they will thin exponentially with mean angular depth, in exactly the same way that in standard space syntax they thin with mean depth. Again, we can reverse the process and suggest that numbers arriving at the node should increase with lower angle to the rest of the system.

In the next section we will formulate the exact analytic method to achieve this, followed in section 4 by showing that improvement occurs consistent both with qualitative and quantitative assessment of systems, and finally in section 5 we will discuss why despite all that we have just proposed, integration to date has been so good a forecaster of human movement. For now, notice that we are still formulating a measure of spatial configuration, not a human cognitive model. We have not moved to the extreme of trying to work out exactly what counts as a turn and what does not, rather we have stayed at the level of spatial analysis, but a spatial analysis which should better forecast people movement.

3 The method

In this section, we discuss how to apply angular analysis to a system. We will first show how it may be applied to axial maps (following Dalton, 2001), and then show how this schema is of the same form as angular analysis as applied to visibility graphs. We will also demonstrate that it may be possible to formulate angular analysis in such a way to overcome the segment problem.

Formally, angular analysis weights any j graph by the angle (in radians) of each connecting pair of axial lines. To calculate angular mean depth, the shortest angular path from every axial line to every other axial line in the system is calculated. The angular mean depth ${L^{\alpha}}_a$ for a line a is the sum of the shortest angular paths over the sum of all angular intersections in the system (not the number of lines in the system for reasons which will come apparent). This is summarised in equation 4:

$$ {L^{\alpha}}_a = \frac{\sum_{b \in V(L)} l_{ab}}{\sum_{e \in E(L)} w_e} \qquad (4) $$

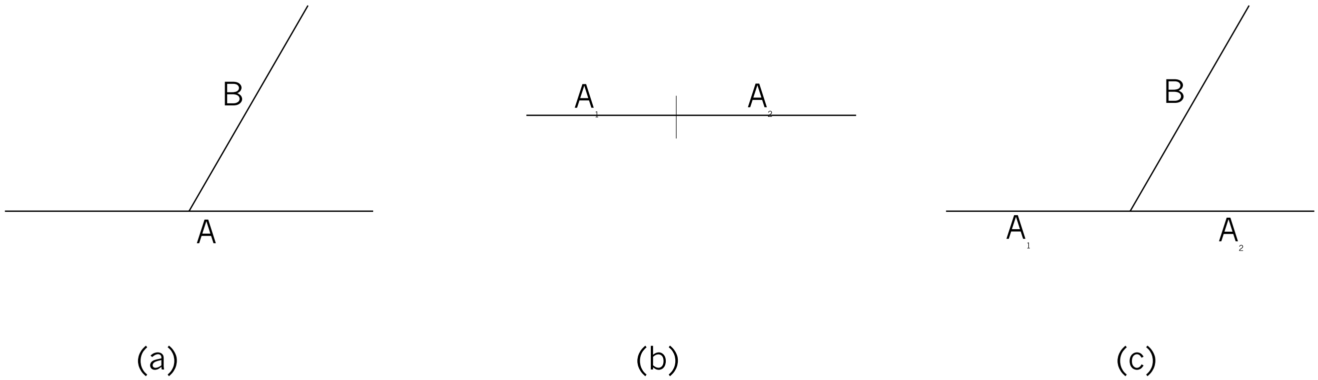


Figure 3: (a) What is the angle between lines A and B? (b) If A is uncut by another axial line, splitting it into two makes no difference to the mean depth of either part. (c) A solution to the cutting problem: split line A into two at its intersection with B.

where l_{ab} is the shortest angular path length between lines a and b, and V(L) is the set of all axial lines in the system, E(L) the set of all edges in the system (i.e., the connections between axial lines in the system), and w_e the 'weight' (i.e., the angle) of each individual connection. Figure 2 shows the calculation of angular mean depth and mean depth for a line in a very simple axial system. Note that resultant figure has no units since it is a ratio of two different angular properties - thus, whether we use degrees, radians or any other measure of angle is immaterial to final mean depth.

Having set up this system, a question arises: what do we do about lines which are cut in the middle, as shown in figure 3a? Is the angle $\pi/4$ radians (45 degrees) or $3\pi/4$ radians (135 degrees)? I would suggest the answer is to split the line into two. First note that due to our earlier stipulation, splitting any line in the system (between any other axial lines crossing it) makes no difference to the angular mean depth of any line in the system, as the total angle has not changed. The two new lines are at 0 radians to each other, so contribute 0 to the total angle of the system, and also 0 to any j graph, as shown in figure 3b. Equally, if a line is split into segments such that an axial line crosses at 90 degrees, there will be no difference to the angular mean depth of any line in the system (as both halves of the line will be at 90 degrees to the rest of the system). So, as opposed to standard axial integration, splitting lines does not affect the overall or individual depth of lines in the system. However, if we split the line in figure 3a at the intersection of A and B, the angle is clearly $\pi/4$ in one case, and $3\pi/4$ in the other, as shown in figure 3c. Thus, line A_2 is more segregated from line B than line A_1 (and vice versa). This means that the system is different as perceived from each segment of an axial line, as would seem logical, and will vary the integration of a line along its length, avoiding the segment problem.

Now we have a formal definition of angular analysis for axial systems, we will extend the definition to visibility graph analysis (VGA). Essentially the definition is the same, but note that in a path from location to location in a visibility graph, it is not the edges that are weighted, but the edges in the *dual* of the visibility graph, as shown in figures 4a and 4b. The result is interesting: by obtaining the dual of the graph, we have returned to an axial-like description. In fact, in a graph which fully describes the system, the edges form the set of all possible lines in the system. Thus the axial lines are simply a reduction of the set of all possible lines in the dual of the visibility graph. Since we must use the dual of the graph to to calculate the angular mean depth from any location we must decide which edges to start from to calculate the depth from a location. The method we use (not the only one) is to take all the edges connecting to the starting location, and calculate the shortest path to all other

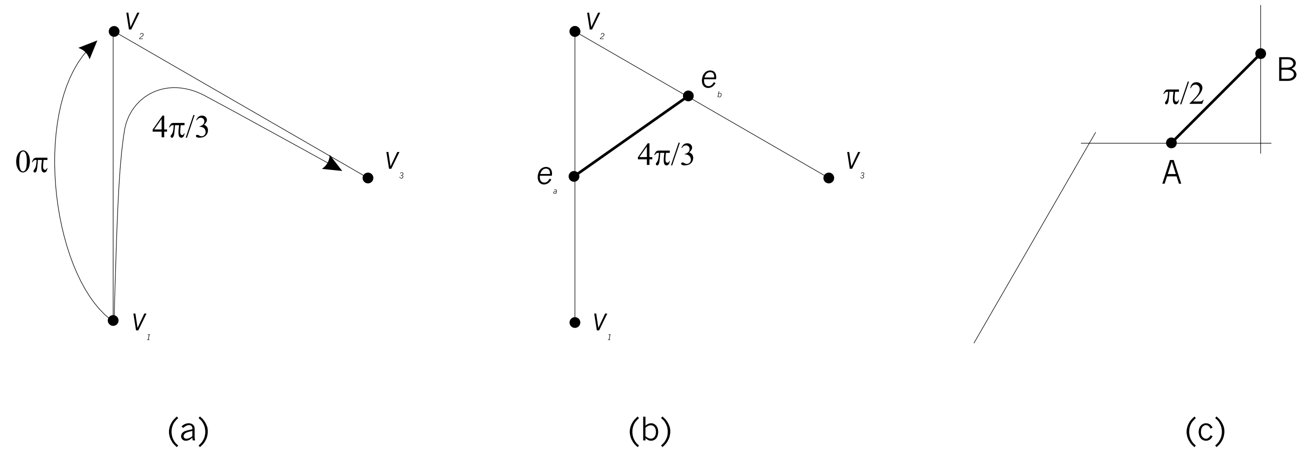


Figure 4: (a) The angular distance from V_1 to V_2 is 0 radians, while from V_1 to V_3 it is $4\pi/3$ radians. (b) The dual of the visibility graph can be used so that the weight between e_a and e_b is $4\pi/3$ radians. (c) This graph as the same form as an axial line graph.

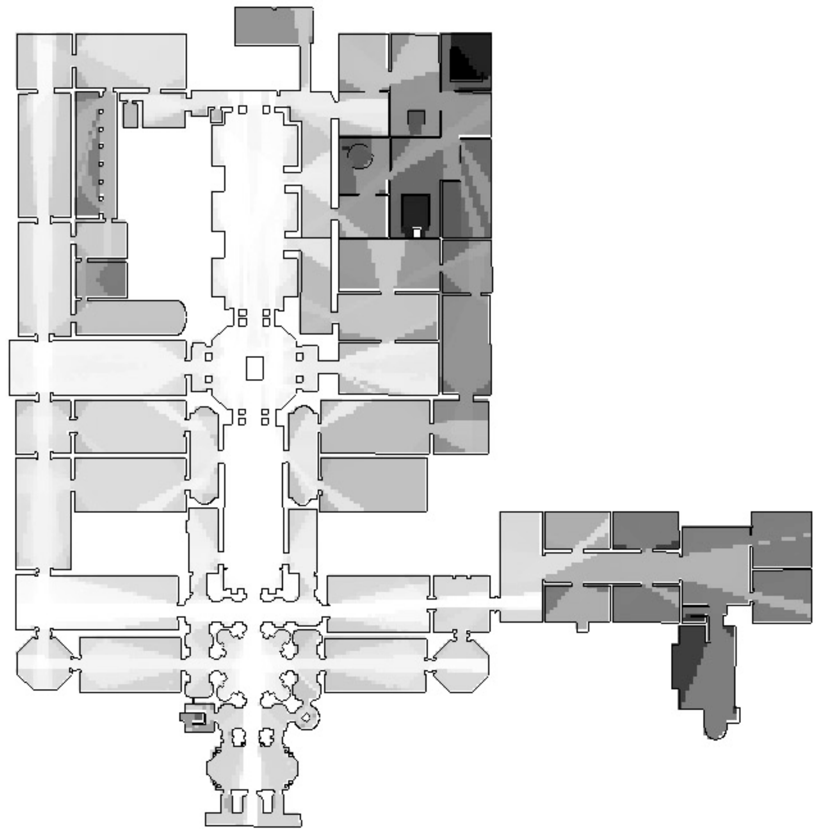
locations by considering the location as reached via the shortest angular path when an edge to it has been found. For example, in figure 4a and 4b, the angular path from V_1 to V_2 is 0 as the edge e_a connects them. Although this description may sound overly complex - after all, we are merely trying to find the path with the minimum (sum) angular devation from location A to location B - the reason for the introduction of the dual is so that we can still can still think of angular VGA as graph analysis, and by approaching angular VGA using graph theory still apply equation 4, as well as any other graph analysis measures we might choose to employ.

4 Application

In this section we present the results of applying angular analysis to both building and urban examples, using angular analysis as an extension of visibility graph analysis (as per Turner, 2000). Technically there is no reason why we should not have applied the methodology using axial analysis, and indeed, other researchers are currently working on angular analysis of axial systems (Dalton, 2001).

4.1 Angular analysis of a building environment

We choose as our building example the Tate Gallery on Millbank. The reason for this choice is that it was used in the original studies on VGA (Turner and Penn, 1999; Turner et al., 2001), and that we have access to good people movement data for the building (Hillier et al., 1996); it is also large enough to provide a variation in depth and non-trivial circulation loops. Figure 5a shows the VGA mean depth calculated in the standard way for the gallery, while figure 5b shows the angular VGA mean depth for the building. The scale is inverted so that it resembles integration: black corresponds to high mean depth (deep j graph) and white corresponds to low mean depth (shallow j graph). The immediately noticeable feature of angular analysis is the smoothness that it introduces. Rather than sharp depth breaks from room to room, angular analysis shows a gradual increase as rooms are entered and angle turned through. One of the striking features of VGA mean depth is that often these sharp depth 'spurs' from corridors into rooms do seem to be genuine features of people movement around the Tate Gallery. However, in reality the people move through these spurs and then into the rest of the room, and this is what appears to be picked up by angular analysis - the spurs still exist, but they are muted on the room side as movement and occupation spread through the room.



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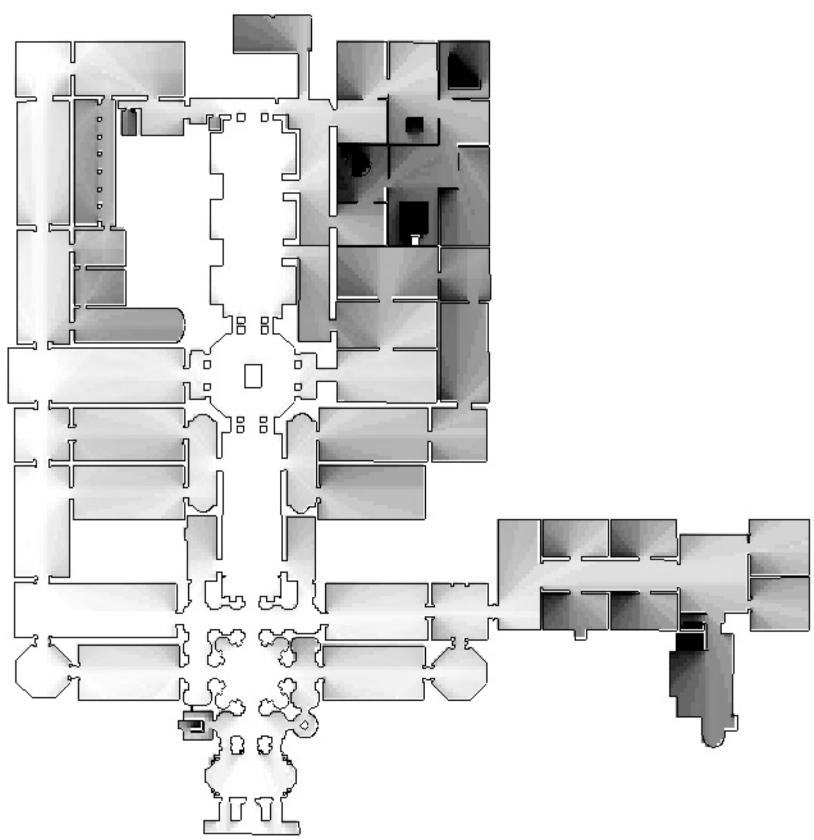
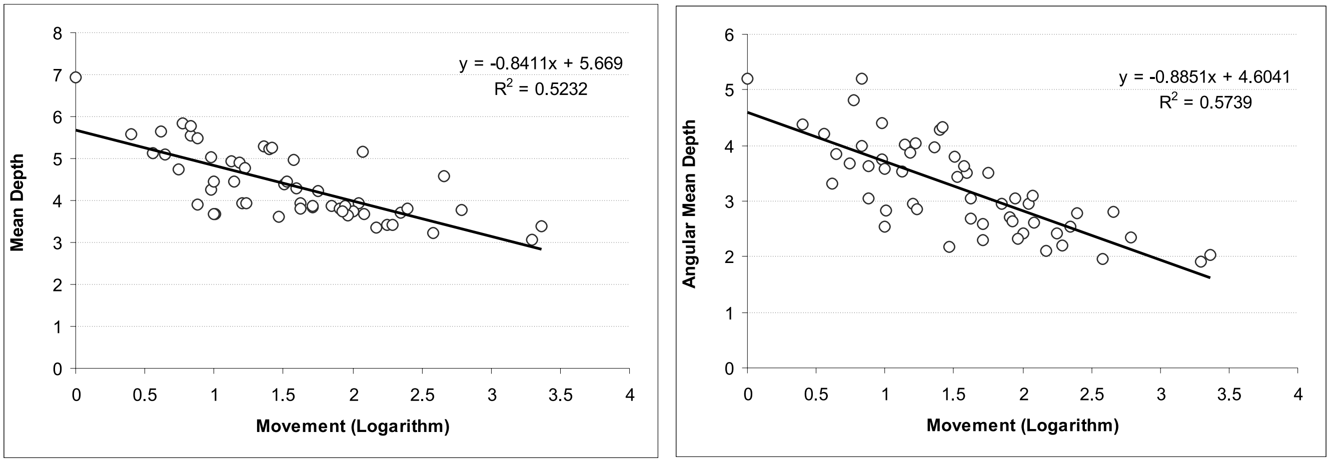


Figure 5: (a) The Tate Gallery on Millbank analysed using VGA mean depth. Low values in white, high values in black. (b) The gallery analysed using angular VGA mean depth.



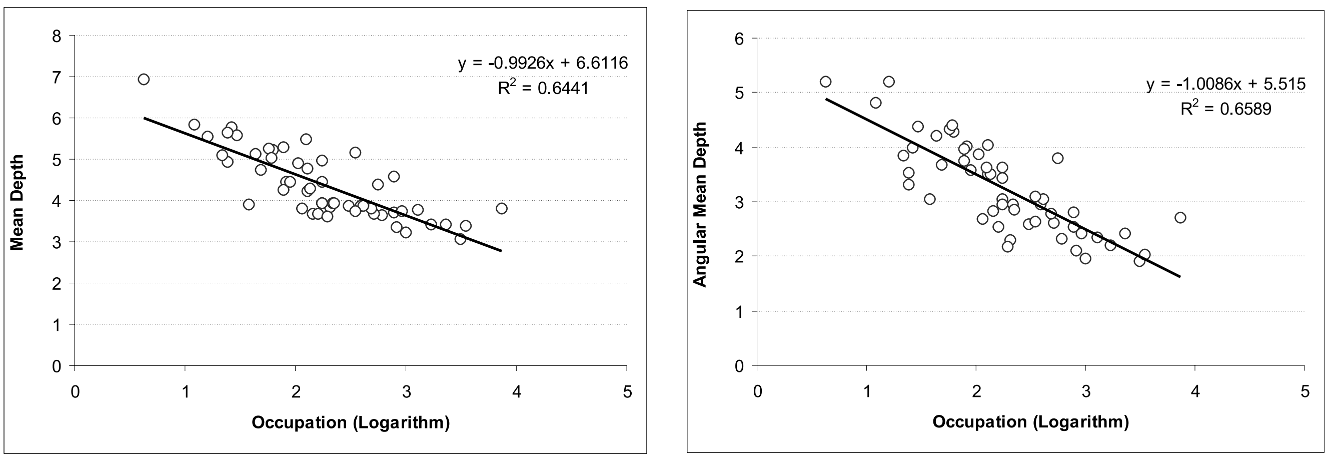


Figure 6 (above): (a) Mean depth plotted against natural logarithm of movement for the Tate Gallery. (b) Angular mean depth plotted against natural logarithm of movement for the Tate Gallery.

Figure 7: (below): (a) Mean depth plotted against natural logarithm of occupation for the Tate Gallery. (b) Angular mean depth plotted against natural logarithm of occupation for the Tate Gallery.

So, angular analysis seems to give a more 'realistic' notion of people movement to spatial analysis, but does it improve on people movement forecasting? Figure 6a shows the standard VGA mean depth against the natural logarithm of room through movement (that is, the average VGA value across the room against the volume of people passing in and out of that room in a given period). As can be seen, $R^2 = 0.523$ (note that Turner and Penn do not give a figure for people movement and instead concentrate on room occupancy). However, if we apply angular mean depth against the natural logarithm of room through movement, as shown in figure 6b, this figure increases to $R^2 = 0.574$, an improvement over standard VGA. Turner and Penn demonstrate that VGA mean depth corresponds better to room occupancy than room through movement. If we apply regression analysis to these data, as shown in figure 7, then there is still an improvement in correlation, though less marked: $R^2 = 0.644$ (standard) to $R^2 = 0.659$ (angular). This improvement still exists if we remove the shop (as did Turner and Penn in their study: the shop is the single room in the Tate which is not gallery space, and a marked outlier in the data): $R^2 = 0.674$ (standard) to $R^2 = 0.695$ (angular).

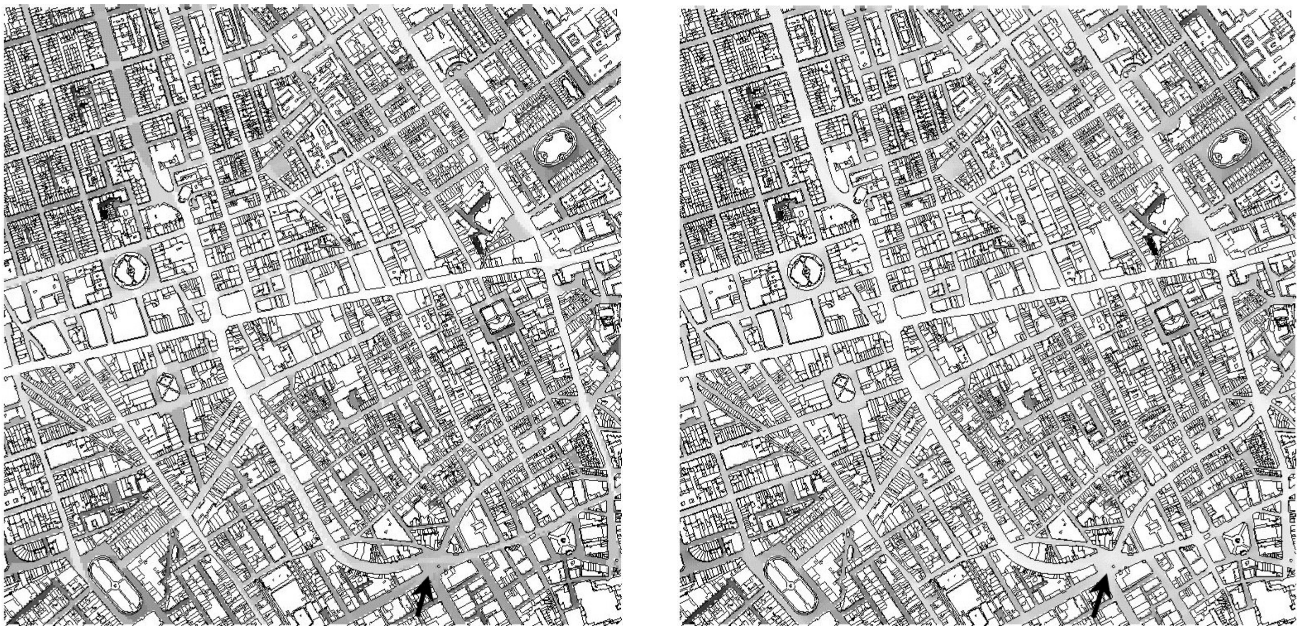


Figure 8: (a) VGA mean depth for a 1.5km by 1.5km area around Oxford Street, London. (b) Angular VGA mean depth for the same area. The arrow marks Picadilly Circus. (Map data reproduced by kind permission of Ordnance Survey *copyright* Crown Copyright ED 281336)

These results would seem to be at least initial confirmation that the argument in section 2 holds. However, the increases seem relatively small. Looking at the layout of the gallery we see that the layout of the rooms, we see that the arrangement is based on a rectilinear grid. There are only a few cases where the choice for a person within the environment is not a 90 degree turn. Therefore, we might well expect the conventional analysis to work almost as well as the angular analysis. Indeed, the inspiration for the technique lies in open plan buildings, where the occupant has more than simple 90 degree route choices. Whether angular analysis will improve on VGA in this context is still open to question and requires further experimentation, however initial analyses by the author are proving encouraging.

4.2 Angular analysis of an urban environment

In addition to analysing building environments, angular VGA may be used to examine urban layouts. Figures 8a and 8b show standard VGA mean depth and angular VGA mean depth respectively, as applied to a 1.5km by 1.5km area around Oxford Street in London. Again, we can note the smoothing of values provided by angular mean depth. In addition though, certain roads known have high volumes of pedestrian movement have been picked out by the new analysis. Note in particular that Picadilly Circus has a shallower mean angular depth than standard mean depth. This is difficult to see directly from the diagrams, but can be ascertained if we compare the deviation of the values from the mean: in angular analysis the locations within Picadilly Circus are 0.66 standard deviations less than the mean, whereas in standard VGA it is 0.13 standard deviations above the mean. That is, in angular analysis Picadilly Circus is very shallow compared with the rest of the system (0.66 s.d.s equates to within the lowest 25% of values), whereas in standard VGA it is slightly deeper than average - i.e., angular analysis has picked Picadilly Circus as an integrated location where standard VGA has not. Both values are probably higher than expected due to edge effect. Unfortunately, we have not completed a regression analysis with people movement in the area as yet, so our assessment of the results can only be qualitative. Even so, there is good reason to be optimistic for the analysis given the results shown here.

5 Discussion

This paper has introduced an enhancement of space syntax methodology called angular analysis, which uses a weighted graph to calculate space syntax metrics.

In section 2 an argument was proposed to explain why angular analysis integration might be an improvement over standard integration in terms of people movement forecasting. The argument was based on the assumption that people movement arises from the mean depth - that people move through a system based on number of turns encountered in that system. We showed that a plausible causal relationship between mean depth and the logarithm of people movement can be formulated. We then suggested that by changing the assumption of route decisions based on number of turns to route decisions based on angle, we would come closer to the probable method of navigation used by humans, as suggested by other researchers. Thus, by replacing 'mean depth' with 'angular mean depth', we proposed a j graph which might be more like the system as experienced by a human, and therefore, according to our causal assumption, would lead to better pedestrian movement forecasting.

In section 3 the angular analysis technique was descibed in detail, explaining how it may be applied to both to axial line maps and visibility graph analysis. We showed that by considering angular deviation in mean depth, rather than binary turns, it is possible to segment an axial line in any way without affecting the overall angular depth of the graph, either when it is uncut by another axial line in the region segmented, or broken by a line at 90 degrees. We then went on to use such segementation to differentiate between different halves of a line when cut by a line not at 90 degrees.

In section 4 the technique was applied to two case studies, using angular visibility graph analysis. For the Tate Galley, Millbank, we showed an increase in people movement correlation, from $R^2 = 0.532$ to $R^2 = 0.574$, when the standard mean depth measure is replaced with angular mean depth. In an urban environment around Oxford Street in London we showed that there was a qualitative improvement when studying an environment, especially in junctions known to be busy (both for traffic and pedestrians) in that area of London.

A question still remains after this study: why does conventional analysis work so well? If we look at the Tate Gallery, one thing is immediately obvious: this building is constructed on a rectilinear grid. As was suggested, perhaps because turns are forced to either 0 degrees or 90 degrees, the average angular values for the rooms are based on choices that are in reality only binary, of turn or no turn. In addition, in the city example, as Hillier (1999) notes for the general case, most turns in the grid are also near 0 degrees or near 90 degrees. Again, as regards people movement, choices are restrained to either straight ahead, or a binary turn. Therefore, as an approach to people movement forecasting, standard integration should work almost as well as angular analysis for many building and urban environments, since the argument made was for choice of non-binary direction decisions; if these direction decisions are instead binary, then the same argument would predict conventional analysis working well. Of course, where angular analysis should come into its own is where these binary relationships no longer exist in buildings and cities - for example around Picadilly Circus in London (marked with an arrow on figure 8) - Regent Street gradually curves in from the West, and Shaftesbury avenue, arrives at an angle from the East. Hence, the increase in integration observed at Picadilly Circus when angular analysis is applied, and perhaps also, the reason for the perception of centrality found in tourist to local alike.

What, of course, angular analysis does not explain is why the city should be a deformed grid in the first place. Nor does it provide, for example, a tool for the spatial description of areas in terms of information content or social use. However, as a tool to help us understand the spatial configuration of a city, or a building, where deformation has occurred or been deliberately chosen by the architect, it should function to help us find how the city or building is still bound together, despite minor discrepancies in joining of corridors or angles of streets. We hope to have shown here that by using angular analysis we might add an extra layer to space syntax, to help better understand spatial configuration and better forecast people movement.

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