Addressing Normalisation in the Pursuit of Comparable Integration

Gillian E. Livesey and Anthony Donegan
University of Ulster, UK

Abstract
Egress complexity provides a non-metric assessment of the egress and access capability of a compartmentalised floorplan. Normalised egress complexity, based on the distribution of non-isomorphic floorplans, enables the relative comparison of configurations with differing numbers of compartments. Recent developments incorporating the Space Syntax philosophy have enabled an assessment of route complexity within noncompartmentalised environments. The latter has led to an examination of the distributions of mean depth, integration and real relative asymmetry. This is discussed in some detail in the present paper. An alternative to the traditional two-part transformation of mean depth, similar to that already employed in egress complexity is proposed.

1. Introduction
For some time the authors have been working in the field of fire safety engineering exploring the egress capability of buildings. Much of this work, in collaboration with others, revolved around the time-required element of the survival inequality

\[ \text{TIME AVAILABLE} \geq \text{TIME REQUIRED}. \]

The time to evacuate a fully occupied building, once the alarm has gone off, is the nearest approximation to this terminology of time-required. Full studies take account of disability factors in the various occupancies, and as a consequence, scenario based models have been developed that can predict these times. It is prudent to manage designs on required times that are at the pessimistic end of the range. This latter constraint raised the question of invariance. In other words, is it possible to get some measure of evacuation capability that is independent of any scenario? The answer was proposed by Donegan et al. (1994) when an attempt was made to measure route complexity - a building specific parameter independent of any scenario. This work has progressed to the stage where it is now possible to compare different building plans and environmental spaces from a complexity perspective. Recent work has enabled the theoretical basis to be applied to other areas beyond fire-safety evaluation such as transport networks, sociodynamics and rural syntax\(^1\). It was while working

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Keywords
Egress complexity, Escape Syntax, normalisation, mean depth and measures of integration

g.livesey@ulst.ac.uk
with the latter that the authors started to become involved with the superficial application of Space Syntax and hence to the motivation for the theme of the present paper.

2. Egress complexity

Egress Complexity (Donegan et al., 1994) is a scenario independent, non-metric methodology that assesses the egress capability of a compartmentalised environment. The measure of capability is effectively the egress route complexity - initially developed to measure the uncertainty experienced by a naïve occupant in pursuit of an exit. More recently, the potential of the methodology for assessing search complexity has been recognised by the authors.

Every building has a latent measure of route complexity that may be computed algorithmically from the layout of the building’s floor plan. The magnitude of this complexity is a “cold” or inert measure of the equivalent topological network of rooms and connecting passageways and is therefore a building specific measure - quite distinct from scenario based measures of time-required modelling. Pollock et al. (1994) discussed an AI procedure for automating the evaluation of egress complexity directly from CAD data.

The mathematical formulation, detailed by Donegan and Pollock (1996), is based on information theory and entropy (Shannon, 1948). Building plans are interpreted as networks, the habitable compartments are represented by nodes and the defined links between the nodes are identified by the arcs - referred to as information steps. Knowledge with respect to egress is gained when an information step is traversed for the first time - thereafter that particular information step makes no contribution to the accumulated knowledge. Specifically, knowledge is not gained if an arc is backtracked. The probabilities of acquiring and of not acquiring egress information in a sweep from any general compartment to a predetermined exit node are determined and the corresponding egress complexity calculated using algorithms produced by Pollock and Donegan (1996).

The practical application of egress complexity was constrained by its initial inability to facilitate building type comparisons. A dimensionally large building and a dimensionally small building can have the same egress complexity (the Russian doll model) but it is also possible for buildings with different floorplans or numbers of compartments to share egress complexity values. For example, the three distinct single exit floorplans shown in Figure 1, each has a calculated egress complexity value of 77.84.
Notice that floorplans A and B have seven compartments and can be deemed equal in terms of egress complexity. Floorplan C however, has eight non-exit nodes and it is therefore impossible to compare the egress complexity of C with that of A or B without some measure of relative or normalised egress complexity. Notice also that apart from a decision point at \( c_6 \) the egress capability of Floorplan C is relatively straightforward. This will be reflected in the normalised calculation. However, before describing normalisation it is necessary to acquaint the reader with some basic technical terminology.

The floorplan network takes the form of a rooted tree (Harary, 1969), a connected set of \( k \geq 1 \) nodes and \( k - 1 \) arcs with no cycles or loops having a single node, its root node, distinguished from the others representing the compartment with access to the predetermined exit node. The networks in Figure 1 have the white root nodes labelled \( c_1 \), the predetermined exits are represented by \( \text{ex} \), and the arrows depict the exit arcs.

Two floorplan configurations are isomorphic if between their sets of compartments, there exists a one-to-one correspondence that preserves adjacency and the relative exit positions correspond. Egress complexity values generated by non-isomorphic floorplans ensure that each floorplan configuration is unique although the complexity values may be repeated for different layout configurations. For example, Floorplan A and Floorplan B, each with 7 habitable nodes, are not isomorphic yet their corresponding egress complexity values are the same.

It was originally assumed that egress complexity values were uniformly distributed. However, generating values for all non-isomorphic floorplans with up to nine compartments revealed the egress complexity distribution shown in Figure 2. The reader will notice that for each fixed number of nodes there is a maximum and a minimum value for the egress complexity. These optimal values follow well de-

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**Figure 1: Three exit floorplans and their corresponding network diagram**

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fined and distinct trajectories, which are characterised by Donegan and McMaster (2002).

The normalisation process takes into account the frequencies with which egress complexity values occur between the minimum and maximum values and results in the distribution of normalised egress complexity values bounded between 0 and 1, shown in Figure 3.

Returning to the example in Figure 1, floorplans A and B each have a normalised egress complexity value of 0.7049 while the normalised egress complexity for floorplan C is 0.0065 concurring with the earlier remark about its simplified egress capability. It is immediate that normalisation produces a means for comparing floorplans with differing numbers of habitable compartments.

3. Wayfinding
The authors have considered “warming” this cold measure of route complexity with the introduction of signage or maps. Passini (1980) describes these wayfinding support systems and the spatial characteristics of a setting as environmental determinants of the spatial problem-solving process known as wayfinding. Butler et al. (1993) and later Passini (1996) however, provide anecdotal evidence from evacuation case studies raising doubt on the association of signage with the necessary spatial information required in finding an escape route. Peponis et al. (1990) comment that the structural properties of building layout can reflect wayfinding performance, and propose a link between the general intelligibility of the built form and a specific wayfinding performance termed search structure. They suggest that people acquire an intuitive grasp of floorplan configuration in terms of depth and integration. This concurs with Hillier and Hanson (1984), who show a relationship between search patterns and the degree of integration of each space.

Escape Syntax (Thompson et al., 1998) also links wayfinding with Space Syntax. It employs Space Syntax techniques in the partitioning of an internal envi-
The union of Space Syntax and wayfinding prompted the notion of combining Space Syntax philosophy with egress complexity. The spatial node concept of axial analysis (Hillier and Hanson, 1984) has motivated the application of egress complexity to the modelling of non-compartmentalisation and spawned considerable interest in the normalisation of mean depth.

4. Space syntax

It is not the purpose of this paper to discuss either the function or philosophy of Space Syntax. Discussion however will focus on around the concept of integration, a measure of relative asymmetry (RA). This is effectively a normalised measure (Steadman, 1983) of the mean depth (MD = summing the depths and dividing by their number) of a system of connected spaces and is calculated using the formula provided by Hillier et al. (1983), namely

\[ RA = \frac{2(MD - 1)}{k - 2} \]

where \( k \) is the number of spaces (internal and external) in the system.

This study considers the depth of the root node from all other nodes. For example, referring to floorplan A in Figure 1, the root node \( c_1 \) is said to be of depth 0. Nodes \( c_2, c_4 \), and \( c_6 \) are at depth 1 because they are directly connected to the root node. Nodes \( c_5 \) and \( c_7 \) are at depth 2 as it is necessary to pass through another node \( c_6 \) to get to \( c_1 \) and so on. Figure 4 shows the distribution of MD values generated from all non-isomorphic systems with up to \( k = 8 \) spaces, the corresponding distribution of RA values is illustrated in Figure 5. The MD distribution is bounded by 1 and \( \frac{4}{2} \), concurring with Steadman’s (1983) findings. The RA distribution is bounded by 0 and 1 for all values of \( k \).

4.1 A Comparison of metric and non-metric modelling

A variety of simulation experiments have been carried out to assess the effect of plan geometry on egress. Non-metric Space Syntax and Egress Complexity results are compared with the metric time-required results produced by EVACNET4 (Kisko...
et al., 1998). The latter is a scenario based interactive computer program employed to model the time-required element of an evacuation process. This system requires a description of a building’s network and the location of occupants at the beginning of the evacuation. Output consists of a detailed statistical description of an optimal (in the sense of minimum time) evacuation of a specific building. Results from this metric model are in terms of time, distance and the number of occupants, contrasting with those of non-metric analysis, which are based purely on the number of spaces and on the connecting geometry.

Seeding arrangements of compartments are examined. For metric modelling purposes, the initial ‘seed’ compartment is a room 7m long and 6m wide, with a fully loaded node capacity of 50 occupants. The seed compartments are linked in either series or parallel to a corridor giving the four floorplans, types a, b, c and d, (labelled by the number of rooms) shown in Figure 6. These arrangements are similar to those used by Tabor (1976) and Willoughby (1975) in their studies of pedestrian circulation.

Hillier et al. (1987) highlight two configurational properties of spatial layouts, depth and choice. This study considers depth from the exit node. The second property, choice, is the existence or otherwise of alternative routes from one space to another. The absence of choice in type a floorplans forces occupants to follow a specific path to the exit. The presence of choice in types b, c and d floorplans creates a wayfinding problem for evacuees to solve.

The various plans are extended up to a maximum of 20 non-exit compartments. These are assessed for route complexity using both raw egress complexity and normalised egress complexity. Mean depth and relative asymmetry values are calculated from the exit node of each floorplan. Room nodes are fully occupied when the EVACNET4 evacuation simulation commences.

Figure 6: Seeding arrangements of compartments
The EVACNET4 output summarised in Figure 7 contrasts with the non-metric results in Figure 5. Floorplan types \( a \) and \( b \), with equal numbers of room nodes produce identical evacuation times. Type \( d \) floorplans take less time to evacuate than type \( c \) floorplans with equal values of \( k \) as the hallway nodes in \( d \) are half the length of those in type \( c \) with the same number of rooms. The relationship between EVACNET4 evacuation time required for the four floorplan types and the number of compartments, \( k \), suggests that time required is a function of the floorplan configuration.

The relationship between room nodes and floorplan configuration, as measured by egress complexity, mean depth, normalised egress complexity and measure of integration, is shown in Figure 8. Given the same number of nodes, networks of types \( c \) and \( d \) form topologically identical trees, so it is not surprising to find their data superimposed in each of the egress complexity and space syntax graphs. In terms of non-zero complexities, taking any vertical cut through the egress complexity and normalised egress complexity graphs, type \( a \) has the least complexity value followed by type \( b \) with the greatest complexities occurring for types \( c \) and \( d \). This order is inverted in graphs of both mean depth and measures of integration. The mean depth plots of all four floorplans appear linear due to linking compartments in either series or parallel. However the relative asymmetry graph show that only the serial linkage of type \( a \) behaves linearly.

4.2 Normalised mean depth
In order to compare systems with different numbers of component spaces however, Hillier and Hanson (1984) point to the necessity of a further transformation process. They suggest that the measure of integration of a system is divided by that of a diamond shape pattern, referred to as the D value (a table of which appears in Hillier and Hanson, 1984: 112), in order to achieve the ‘real relative asymmetry’ or RRA of the system. For example, Figure 9 shows all nine possible non-isomorphic systems.
with 5 component spaces together with their mean depths from the root nodes and corresponding relative asymmetry values. The RA values are then divided by the D value for systems with 5 spaces, 0.352 (Hillier and Hanson, 1984), to achieve corresponding RRA values. While RA values range from 0 and 1, the RRA values range from 0 to 2.84.

Hillier and Hanson do not quote D values for \( k \leq 4 \) spaces. For systems with \( k \geq 5 \), RRA values are bounded by 0 and \( D^{-1} \). Figure 10 illustrates the distribution of RRA values for systems with \( 5 \leq k \leq 8 \). The underlying shape of the distribution for each \( k \) value remains unchanged from the MD and RA distributions in Figures 4 and 5, only the bounds of the distribution are altered as a result of this secondary transformation process.

Other secondary procedures have been proposed. Krüger (1989) has suggested the use of alternative transformations on integration values, based on the assumption that the depth of nodes on a standardised axial map is normally distributed. These transformations also involve dividing a system’s measure of integration by the RA value of a standardised axial map. The standardised RA values for systems with more than 3 spaces tend towards 0 as \( k \) increases and are bounded above by 1. As with Hillier and Hanson’s RRA values, Krüger’s transformed MD values are bounded below by 0 and above by the inverse of the standardised RA values. Tekleburgh et al. (1993) suggest a logarithmic transformation on the total depth of a system, but this method cannot produce values for all axial maps.

In this paper, a new method of mean depth normalisation is proposed. The normalisation technique is similar to that already employed to achieve normalised egress complexity values. This mean depth normalisation method incorporates distributions of values generated from all non-isomorphic configurations of \( k \) spaces. Its resulting normalised mean depth values are bounded by 0 and 1, and relative comparisons of systems with different numbers of elements can be made without the need for a secondary transformation.

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**Figure 9:** Non-isomorphic systems with 5 component spaces and their corresponding Mean Depth, Relative Asymmetry and Real Relative Asymmetry Values

<table>
<thead>
<tr>
<th>System</th>
<th>MD</th>
<th>RA</th>
<th>RRA</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i)</td>
<td>2.5</td>
<td>1</td>
<td>2.84</td>
</tr>
<tr>
<td>(ii)</td>
<td>2.25</td>
<td>0.83</td>
<td>2.37</td>
</tr>
<tr>
<td>(iii)</td>
<td>2</td>
<td>0.67</td>
<td>1.89</td>
</tr>
<tr>
<td>(iv)</td>
<td>1.75</td>
<td>0.5</td>
<td>1.42</td>
</tr>
<tr>
<td>(v)</td>
<td>1.75</td>
<td>0.5</td>
<td>1.42</td>
</tr>
<tr>
<td>(vi)</td>
<td>1.5</td>
<td>0.33</td>
<td>0.95</td>
</tr>
<tr>
<td>(vii)</td>
<td>1.5</td>
<td>0.33</td>
<td>0.95</td>
</tr>
<tr>
<td>(viii)</td>
<td>1.25</td>
<td>0.17</td>
<td>0.47</td>
</tr>
<tr>
<td>(ix)</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

**Figure 10:** Distributions of Real Relative Asymmetry for systems with up to 8 component spaces

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Addressing normalisation in the pursuit of comparable integration
The RA and RRA transformations appear to assume that MD is uniformly distributed. For each value of \( k \) (spaces) however, the MD values for all non-isomorphic systems results in a positively skewed distribution as shown in Figure 11.

Let \( A_k \) non-isomorphic systems with \( k \) spaces generate \( n \) MD values, and let each of these \( n \) values occur with frequency \( f_i \geq 1 \), \( 1 \leq i \leq n \). The monotonic sequence of distinct MD values for \( k \) is given by \( \{ MD(k)_1, \cdots, MD(k)_n \} \), where \( MD(k)_i < MD(k)_{i+1} \), \( MD(k)_1 = 1 \) and \( MD(k)_n = \frac{k}{2} \). Normalised mean depth, NMD, for a system with mean depth \( MD(k)_j \), \( 1 \leq j \leq n \), is calculated by dividing the sum of ascending mean depths up to, but not including, \( MD(k)_j \) by the sum of all mean depths generated by the \( A_k \) non-isomorphic systems with \( k \) spaces using the following formula:

\[
NMD(k)_j = \frac{\sum_{i=1}^{i=A_k} MD(k)_i - \sum_{i=j}^{i=A_k} f_i MD(k)_i}{\sum_{i=1}^{i=A_k} MD(k)_i}
\]

which simplifies to

\[
NMD(k)_j = 1 - \frac{\sum_{i=j}^{i=A_k} f_i MD(k)_i}{\sum_{i=1}^{i=A_k} MD(k)_i}
\]

The MD values are weighted according to the frequency with which they occur, with minimum NMD = 0 for all values of \( k \) and maximum \( NMD(k) \) tending to 1 as \( k \) tends to \( \infty \).

This revised method, which can be applied to both buildings and settlements, captures the initial integration bounds, 0 and 1 and permits the direct comparison of spatial structures with different numbers of elements. The non-isomorphic systems with 5 spaces in Figure 9 generate the NMD values in table 1.

<table>
<thead>
<tr>
<th>System(i)</th>
<th>(i)</th>
<th>(ii)</th>
<th>(iii)</th>
<th>(iv)</th>
<th>(v)</th>
<th>(vi)</th>
<th>(vii)</th>
<th>(viii)</th>
<th>(ix)</th>
</tr>
</thead>
<tbody>
<tr>
<td>NMD</td>
<td>0.84</td>
<td>0.69</td>
<td>0.56</td>
<td>0.34</td>
<td>0.34</td>
<td>0.15</td>
<td>0.15</td>
<td>0.06</td>
<td>0</td>
</tr>
</tbody>
</table>

Figure 12 shows the distribution of NMD values generated from all non-isomorphic systems with up to 8 spaces.

Unlike other MD transformations, the NMD distribution differs in shape from that of MD and echoes Hillier’s (1996) observation that buildings “tend to become relatively less deep as they grow”. Mean NMD values are lower than those of the RA and RRA distributions, for example, when \( k = 8 \) the mean NMD is 0.39, compared with a mean RA of 0.45 and mean RRA of 1.371.
Software is currently being developed by the authors to enable all possible non-isomorphic network configurations and their mean depths to be generated together with the computation of normalised mean depth for individual spatial networks.

Notes

1 Rural Syntax - BArch(Hons) Thesis by T.L. Donegan, University of Dundee, Duncan of Jordanstone College (2000).

References


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