Storing directionality in axial lines using complex node depths

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Abstract

This paper proposes that in order to implement an angular-based choice[1] algorithm it is first necessary to implement a new type of *depth* definition. Such a depth algorithm would not only calculate the 'minimum' angular depth from any origin to any destination (as per Dalton, 2001) but specifically stores the depth as *complex number*, which additionally represents the cumulative angle that facilitated that particular minimum angular depth calculation. By using such a representation it becomes possible to compute the unique angle of intersection of any two axial lines, where the starting-direction of a hypothetical individual travelling from one axial line to another is known. This paper concludes with the suggestion that the use of complex number depths (namely depths that have a real and imaginary component) is an interesting and valuable extension of the concept of depth; originally depth could take only an integer value, this was then extended to a real numbers (angular depth) and finally has been extended once more by utilising complex numbers. The use of such an algorithm, as will be described in this paper, to calculate *complex depth* can then be used to compute true angular depth and hence *angular choice* for any given axial system. This paper will present the proposed algorithm and new measure in full.

Keywords
angular depth, fractional integration, choice measure, directinality

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1. Introduction

In recent years, algorithms that employ the concept of *weighted*, *non-integer* step-depths have begun to emerge as an interesting variant of the topological-based measures traditionally used in space syntax research such as integration. One example of the use of a weighted graph was presented in the author's earlier paper on fractional integration (Dalton, 2001). A limitation of the current algorithm used to compute weighted, angular depth is the lack of prior knowledge concerning the angle of incidence between two lines; two angles may be calculated between any two intersecting lines, ß and ∂ such that (2 x ß) + (2 x ∂) = 360°. The algorithms used to calculate angular depth made the essential assumption that the smaller angle should always be taken account of and the larger angle disregarded. This was considered to be a necessary step taken to simplify the algorithm. Theoretically this was justified on the basis that since angular integration, like integration, considers all journeys from everywhere to everywhere else that this assumption would balance out over all journeys. However, intuitive observations of typical test cases suggest that one angle often appears the more likely for any given journey. Furthermore, from an algorithmic perspective, the computer is not actually constructing a series of hypothetical journeys though a map but rather is moving though an abstract graph representation.

The problem of a mismatch between algorithmic and common sense conceptions of journeys, is exacerbated when considering possible implementations of algorithms such as *choice* which are implicitly journey-based. The choice algorithm/measure has always had a strong attraction to researchers due to the fact that it is based upon a hypothetical individual moving through the axial map/graph in a specific manner and hence is easy to both comprehend and conceptualise. Such individual journeys are then simply aggregated throughout the system, resulting in a choice value for each axial line. However, since correlations between the measure choice and pedestrian movement observations have failed to improve upon correlations obtained with algorithmically simpler measures such as integration then the simpler measures have tended to prove more popular. This paper proposes that by introducing the angle between any two axial lines as a pertinent factor in route selection, then the new measure of *angular choice* might result in improved correlations with pedestrian movement and possibly explain why choice has failed to out-perform integration despite its clearer situated individual model.

2. Angular or fractional analysis

A previous paper (Dalton, 2001) introduced the concept of angular depth as a method to solve a range of problems in the construction of the axial map.

"Fractional Angular Analysis works by defining a fractional analysis where the angle of incidence is 1.0 where the axial lines are at right (90 degree) angles. Lines that are parallel and intersect have fractional distance of 0.0… From this, lines that are nearly parallel have low fractional distances [from one another]. We would expect this kind of analysis to make long meandering streets [comprised] of many axial lines to become stronger integrators [than would be the result if using traditional axial analysis]. Equally making one right-angled turn might well mean a strong increase in 'distance'… We now can build a network as in the traditional manner, except here the graph will have steps which are not 'integer' numbers (1, 2, 3) but rational or fractional numbers)." (Dalton, 2001)[2]

While the fractional method described above facilitates numerous benefits, both practical and theoretical, a number of questions arise from this method, which need further evaluation. One natural question raised by Asami et al. (Asami, 2002) in their paper on fractional angular analysis, is the question regarding obtuse angles and the implementation of angularity with respect to related measures such as choice.

The question is posed of why obtuse angles are avoided by fractional angular integration. Consider Figure 1, which is a representation of an axial system, marked with specific locations (a1 and a2) and axial lines (a1—a2, b, c, d). If we first imagine a pedestrian starting at point a1 and walking towards line b, then clearly the angle the pedestrian must turn, as they leave line a1—a2 and start moving along line b is 60°. If that same person were to start at point a2 and also turn to walk along line b then the angle they would turn through this time would be 120°. There has been a criticism at the convention used in fractional angular depth analysis of finding the smallest angle (60° in both cases for a1—a2.b and a1—a2.c) between two lines for the depth-weighting function. Intuitively it seems as if it should be both possible and easy to include angles greater than 120°. If the axial line labelled b in Figure 1 did not exist in the fractional system, then our imaginary pedestrian would have to turn 120° if travelling from point a1 to line d (via line c). The fractional algorithm assumes the angle rotated through must be 60° (the smaller of 120° and 60°). The algorithm would then apply a weighting of 0.6667 to this edge in the associated graph instead of the more natural angle of 120°, which would give a weighting of 1.3333.

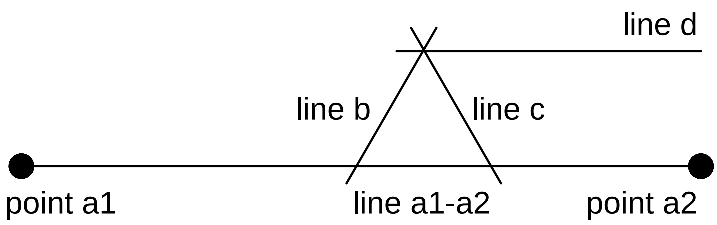

Figure 1. A Sample Axial System.

Does this omission to consider obtuse angles of incidence negate the mathematics or indeed the functionality of fractional angular integration? It is suggested that this is not the case. To understand why angles are limited to the smallest angle £ 90°, consider a pedestrian moving along the axial line a1—a2 from the point a1 and navigating to line d on the axial map in Figure 1. Intuitively we would imagine they should select line b in preference to line c (i.e. that they probably started at point a1). This intuition is substantiated by observational data from (Conroy Dalton, 2003). Those who object to limiting fractional depth values to 90° or less fail to consider the reverse case of a pedestrian moving from point a2 on the axial line a1—a2 towards d. In this case the angle moved through from the line a1—a2 via line c is now also 60°. Now consider the third case, where the imaginary pedestrian begins their trip from midway between the intersections a1—a2.b and a1—a2 .c. Which route would our intuition suggest in this case? This thought experiment has been included in the paper to demonstrate that visually we are often deceived into reading diagrams from right-to-left and we fail to consider the other possible left-to-right case. The algorithm, which calculates fractional depth, however, makes no such assumptions.

Where do pairs of *extremely* obtuse and acute angles of intersection (i.e. angles of intersection least similar to a 90° turn) occur in urban systems? It could be suggested that these pairs of angles of intersection often occur in sequences of axial lines, forming meandering routes through the urban fabric; these lines frequently serve as arterial or center-to-edge routes (Hiller, 2001). I would also suggest that when considering these meandering routes, comprising of a number of axial line segments connected shallowly, that we imagine them to be comprised of acute rather than obtuse angles of incidence. That is to say, that we perceive that we would make only small changes of direction were we to be traversing such a sequence of axial lines. It is for this reason of *environment-perception*, that it seemed logical to give preference to acute angles over obtuse ones and hence the resultant 90° limit originally made in fractional angular depth calculations. The following two examples show how our perception of a diagram or a system can differ widely although computationally they would be identical.

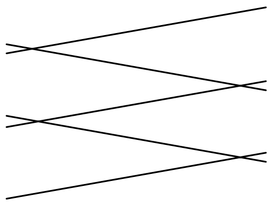

Figure 2. A Hypothetical Switchback.

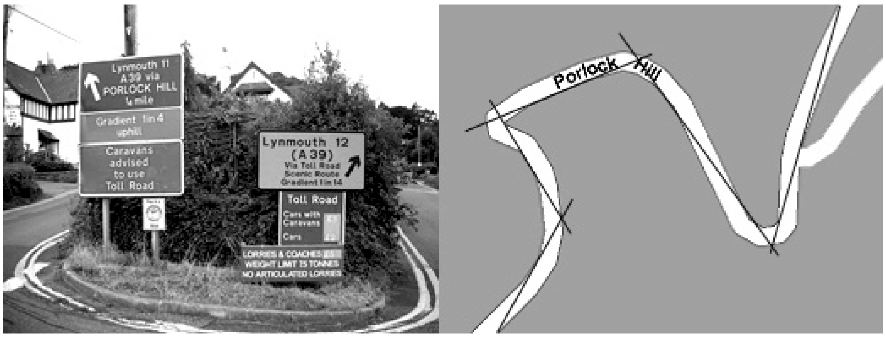

Figure 3. Porlock Hill, Somerset

The case for limiting angles to less than 90° appears to fail in the case of a switchback[3] as in Figure 2. Such an axial configuration as this might be occur in a village on a hill, such as the famous Porlock Hill, outside the village of Porlock (Figure 3). Surely this is an example where the algorithm should allow for angles greater than 90°. Yet by simply rearranging the relative positions of the same axial lines that appear in Figure 2, *but* crucially not altering the angles between the lines nor their topological order we can produce the shallow-angled, meandering street illustrated in Figure 4. This configuration begins to resemble the meandering, arterial roads described in the preceding paragraph.

If the above three figures are identical in terms of fractional angular analysis, are we therefore condemned to never being able to accurately represent the axial configurations in Figures 2 and 3 as being distinguishable from the example of Figure 4? This paper provides a solution to this dilemma by arguing that we are in fact limited by calculating only one value to each axial line.

Figure 4. The Same Set of Axial Lines Perceived as Being Connected Shallowly.

3. Axial depth from an origin

An interesting question to ask, in relation to the dilemma posed in Section 2, is whether we are therefore committed to having only a single axial depth from any given starting point in a graph? One approach to this question is purely algorithmic. Let us consider a simple implementation of a depth finding algorithm (More efficient algorithms such as Dijkstra[4] exist (Gibbons, 1994) but they all share the same flaw, that of being ignorant of 'direction'). In the next paragraph, we can briefly examine the pseudocode[5] for a recursive implementation of a simple depth finding algorithm.

This implementation is successful if all depths are set initially to infinity, except for the starting node, which must have depth of 0 or 1. For a large, ringy

```
recursivedepth( startingnode , depth )
{
        for each node n from startingnode
                if depth[ n ] > depth +1 then
                        depth[ n ] = depth + 1
                        recursivedepth( n, depth + 1 )
end if
        next node
}
```

system a given node may actually be visited several times, with its depth being reduced to its minimum at each visit. Notice that it is typical of such implementations that only the depth of a node is stored by the algorithm and that there is a tendency to ignore the source-path of the depth. Such an approach is necessary in the case of integration as used in traditional space syntax analysis. For example, in Figure 5, line d has a depth of 2 steps from line a and lines b and c both have depths of 1. Which, therefore, is the 'parent' of line d's depth, line b or line c?

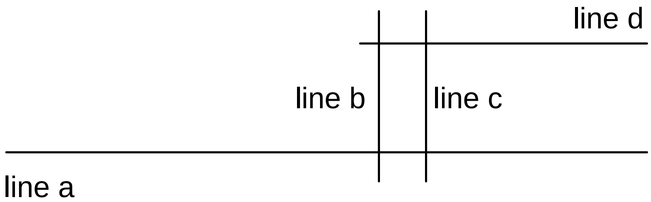

Figure 5. Another Sample Axial System.

4. Choice

Choice is a measure that has existed for some time. The original concept was to create a graph measure, which had a more explicit link between the conceptual behavioural-model of the pedestrian and the larger scale effects. This is similar in some respects to the origin destination matrices used in transportation models except every axial line is simultaneously an origin and a destination. Although it has been demonstrated empirically that there is a strong correlation between integration values of axial lines and observed aggregate patterns of pedestrian flow, there is still no unequivocal cognitive model that explains how the micro-scale decisions of pedestrians could give rise to such a correlation.

Conceptually the measure known as 'choice' is akin to stationing observers on every axial line in the city. Imagine a person starting at an axial line and then walking by the shortest route to another axial line. This process is repeated until our fictitious subject has journeyed from every axial line to every other axial line in the system. While they were performing this Herculean task, an army of observers posted on each axial line were busy tallying up the number of times the subject was observed to have passed them. If two or more equally short routes exist between a pair of origin and destination lines then our pedestrian bisects himself forming two half-pedestrians and each half travels down both streets simultaneously. Our half-persons further sub-divide at any subsequent equally short route-sections. In fact, this resembles a large-scale quantum experiment in which the pedestrian becomes a probability field passing though the urban system. Currently, the only implementation of the choice algorithm forms the software application written by the author, called 'James Choice[6]'.

The choice measure can be expressed algorithmically as the following:

```
For each axial line in the system startLine
    For each axial line in the system which is not startLine called endLine
        Find the shortest distance from startLine to endLine

            being with Value of 1.0 and startLine as thisLine

            if there are two or more equally short routes from thisLine To endLine
                add the Value to the cumulative total for this line.
                divide Value equally between them and use this fraction value.
                repeat for each segment closer to endLine
            if thisLine is endLine stop
    next endLine
Next startLine
```

Computationally choice is far more time consuming to compute than integration or even fractional integration. As the size of the system grows, the whole process becomes much more intensive to compute[7]. Informally, choice has been calculated for a sample of smaller systems and there appears to be no significant improvement over integration. This opinion is largely hearsay and there have been, as yet, no substantial experiments to establish the correlation improvement or reduction of choice compared to integration analysis.

The appeal of choice is the clear and explicit link between space and the behavior of a pedestrian. The fact that choice does not predict patterns of urban pedestrian flow any more accurately then integration (either traditional or fractional) confounds those who might wish to promote a more simulation-based approach to urban analysis (By, for example, introducing the concept of trip origins and destinations). However, this paper asks the question of whether there might be some aspect of the traditional choice algorithm that, if adapted for use in angle-sensitive systems, might promote choice over integration in terms of pedestrian movement prediction.

The introduction of a *fractional-style* processing gives a new direction with which approach the original choice algorithm. If, upon reaching a junction of two equally short topological routes from a line A to a line B, the numbers of people observed to take one route are divided by the ratios of the respective angles, then the new *angular choice* algorithm might more comfortably reflect our intuitive route choice selection. In the case of Figure 1, the fractional choice algorithm for a journey from line a1—a2 to line d would provide higher values of angular choice to line b than to line c, as an intermediate step in the calculation. Compare this to the traditional version of choice, which would assign a value of 0.5 to lines b and c. While a fractional approach may or may not improve the correlation with observed pedestrian movement[8] this approach does marry two naturally affinitive partners: choice and angular integration. However the approach of implementing choice in an angle-sensitive system does reinforce the directional problem outlined earlier in this paper.

5. Resolving the directional problem

In order to resolve the problem of taking direction into account, two solutions naturally occur. The first of these is to segment the axial lines. This has been partly attempted by Asami et al. (Asami, 2002). This removes the whole problem of establishing which angle is formed by an axial line crossing another line, as approached from a given direction. The segmental approach, while quite viable, creates new problems such as how to aggregate the values of the various line-segments. There are benefits to having a single value of integration (fractional or otherwise) for a single axial line. In a segmented system, which of the segments would best represent the parent axial line or would the integration value be the mean of all of a line's child segments?

The second solution is the subject of this paper. Normally depth is stored as an integer (0, 1, 2, 3, 4 etc) in the case of traditional integration and as a real number (0.0, 0.000001, 0.3332, 2.2322132 etc) in the case of fractional analysis. The solution of angular choice depends on storing depth as a mathematical complex number with both real and imaginary components.

Complex numbers were invented by William Rowan Hamilton in 1843. The original intent was to provide a mechanism to calculate the square root of a negative number. A complex number consists of a real part, which resembles our usual number system and an 'imaginary' part which represents multiples of i, where i stands for the square root of minus one. In essence, a complex number exists only partially on the real number line. A complex number might be written as,

$$\mathbf{(45 + 12i)}$$

Where i2 = -1. In this context, we can use complex numbers to represent a vector or a direction. The mathematics of complex numbers permits us to add, subtract, multiply and divide complex numbers in the same manner that traditional mathematics allows us to operate with non-complex numbers (for example, calculating step-depths in traditional axial maps). Using a complex representation, we can store the depth of a line as a complex value and hence retain a 'direction' associated with that particular depth.

By using complex numbers for storing the step depth from an origin, we can maintain the direction that facilitated the minimum depth calculation. In other words, we have a mechanism to preserve the 'direction' (as a vector) from the starting node; this gives a direction with which we can combine an angle-sensitive system in which depth may be more than 90°. Another way of phrasing this is that the use of complex numbers permits the algorithm to maintain a sense of direction though the graph of the axial system.

Algorithmically it might function as follows. Let the variable *ddepth* (directional depth) be a complex number that represents direction. The line begins in both directions for each direction *ddepth* from starting node *n*.

```
recursivedepth( startingnode , direction ddepth )
{
            for each node n from startingnode
                        if depth[ n ] > depth + angle(ddepth, n) then
                                    depth[ n ] = depth + angle(ddepth, n)
                                    recursivedepth( n, depth + angle(ddepth, n) )
end if
            next node
}
```

The key component of the above function is the computation of the angle between the direction of the axial line (node n in the graph) and depth *ddepth*. By storing the axial line as a vector, the operation of the dot product will provide the increase in angular depth. This increase in angular depth and the resultant direction can be stored in the complex-number value of angular depth, and is passed on down. Consider the case of performing these calculations for Figure 1.

The calculation first begins from point a1 and considers movement in the left-to-right direction. The depth of line c is considered and the change in angle is more than 90° (although it would be less than 90° in the right-to-left alternative starting point, a2). The change from line a to line c (the intersection of the two lines) is then stored in the value for line c as both the depth and the angle (pointing towards the top of the page, in this case). The depth of line b is computed in the same manner. At the point of transition from line c to line d the angle is unambiguous. When compared the angle from point a2 to line b to line d the value is clearly less.

Upon repeating this process with the alternative right-to-left direction (a2 to line c to line d) the resulting value is now smaller and the route a2 to line b to line d is the larger. This ability to start a hypothetical route *from both directions* clearly permits all possible depth calculations. Each starting axial line would then have two depths associated with it, which may or may not be identical depending on the configuration of the lines. This difference, which is dependant upon the starting direction, may or may not also serve to be an interesting symmetry measure of the system.

6. Conclusions

By storing the intermediate-stage step-depths as complex numbers, it is therefore possible to simply extend the current generation of space syntax software to be able to store the associated directions of movement. This process more naturally reflects the concept of a 'path' or a 'trail' though an urban system than any of the current measures or algorithms are capable of doing. Further work needs to be performed to merge this measure with other path-dependant algorithms such as choice to the concepts of angular fractionally. In addition to this, some work remains to be done in order to implement this computational approach in a user-ready software package. Such a software program would allow fractional integration values and fractional angular choice to be computed with acute angles. If such a piece of software could be created, then comparisons would need to be made between choice, complex choice, integration, fractional angular integration, complex angular integration and real world observations in order to determine which might represent empirical pedestrian observations most accurately. Finally, further research would need to be conducted to begin to broach the problem of how to compute a relativization formula in the domain of complex-depths.

Notes

[1] See section 5.0 for a definition and algorithm of choice.

[2] This process was implemented by a program called MeanDA (Mean Depth Angular).

[3] A road, trail, or railroad track that follows a zigzag course on a steep incline.

[4] Djikstra's algorithm (named after its discover, E.W. Dijkstra) solves the problem of finding the shortest path from a point in a graph (the source) to a destination. It turns out that one can find the shortest paths from a given source to all points in a graph in the same time, hence this problem is sometimes called the 'single-source shortest paths' problem.

[5] Pseudocode is a detailed yet readable description of what a computer program or algorithm must do, expressed in a formally styled natural language rather than in a programming language.

[6] The title of the software 'James Choice' was intended to be a whimsical pun on the name of the author 'James Joyce' because of the myriad of narrative paths within his novel Finnegans Wake.

[7] Choice is an n3log(n) process in comparison to integration and fractional integration which are both n2log(n) calculations, where n is the number of lines in the system.

[8] Such empirical research has yet to be conducted but would be a fruitful area for future research into this topic.

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