Methods and principles of police deployment within the “chaotic” urban structure

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Abstract
The first part of the following paper focuses on the discovery of fractal dimension within the all too chaotic structure of Tokyo streets, using the relation between the length of the streets and the area that they occupied. Hack’s formula for the analysis of river basins was utilized, and the presence of fractal dimension was discovered in the high-density street zones branching within a limited territory. The second part of the paper explores how the measurement of fractal dimension of irregular street patterns can contribute to the development of more precise methods of the strategic police deployment within the “chaotic” urban structure.

Keywords:
“chaotic”zones with irregular streets, street order, area order, police deployment, map of valuable objects distribution, map of the fractal zones

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1. Introduction

During various kinds of estimation of irregular urban patterns, the method of oversimplifying the real situation has been widely distributed. Zones with the irregular street patterns have been neglected and this usually been masked by the method of artificially adding error to the result. With the discovery of the new non-linear science at the end of the XXth century (Chaos Theory, Fractals, etc.), it became obvious that the tiny differences in input could quickly become overwhelming differences in output (Butterfly Effect) (Gleick, 1987). Urban models based on the oversimplification of the irregular street pattern structure might appear to be misleading, and even dangerous.

With the development of Fractal Geometry, Benoit Mandelbrot introduced new tools for analyzing and measuring irregularity (Mandelbrot, 1982). It has become possible to describe a jagged, fractured, and twisted family of shapes by the mathematical language and, consequently, develop more precise methods of measurement of urban structure. Mandelbrot showed that “chaotic” objects with irregular structure form a class of objects in which length, area, or volume in one dimension had no meaning. Irregular lines, areas, and finite volumes according to Mandelbrot exceed their topological dimensions 1, 2, 3, and can be described by the fractional dimension (fractal dimension) that lies somewhere in between 1 and 2 dimensions for line, 2 and 3 for a plane, etc. Fractal dimension shows the power of irregularity, the movement, that phenomenon distinguishes straight line from the indent one. Irregular objects consist of parts similar to the whole in some way.

There is a widespread theory that as soon as nature began organizing itself by means of simple physical laws, it repeated, with infinite patience, self-similar patterns within the irregular structures. New world view on the development of irregular complex systems, and the discovery of the self-similar patterns within them formed first in the laboratories of Mathematicians and Physicists, spread widely and became the interdisciplinary science. Since the popularization of fractals during the last decade, “chaotic” patterns were found in nature: coastlines, clouds, tree, the functioning of human lungs, blood circulation, etc (Feder, 1988).

Fractal methods of analysis penetrated the field of architectural and urban analysis also. Charles Jenks, in his book “Architecture of the Jumping Universe”, discusses the creation of the new language of form based on fractals, self-organization, and self-similarity (Jencks, 1997).On a larger scale, the research on what is density and how it can be measured was performed by M. Batty and Paul Longley. They identified the new way of thinking about the city’s shape, order, and form by creation of useful analogies between fractal processes in physical systems: electro-deposition, viscous fingering, and crystallization; research on the methods of adaptation of the “Diffusion Limited Aggregation” and the “Dielectric Breakdown” mode to the simulation of city growth and form (Batty, 1991). They introduced the new method of computer simulation of the irregular form of cities in terms of land use development (Batty and Longley, 1986) and showed the way of measurement of urban boundaries and edges (Batty and Longley, 1988).

In our research on the defining of the fractal dimension of the “chaotic” urban patterns, we made use of Hack’s formula which he developed during his study of fractal dimension of rivers in Virginia and Maryland (Feder, 1988). Following Hack, who used the ratio between the longest liver and its drainage area, we took the relation between the length of the streets and the area they occupied as the principle factor of our investigation.

2. The discovery of the fractal dimension within the “chaotic” street pattern

For circle, squares, equilateral triangles and other polygons the ratio between the perimeter and the square root of the enclosed area,

$$\rho = (\text{Perimeter})/(\text{Area})^{1/2} \tag{1.1}$$

is independent of the size of the polygon. The ratio ρ is the same for all closed curves of the same shape, and is $\rho=2\sqrt{\pi}$, 4, and $6/3^{1/4}$, respectively.

For a collection of similar islands of different sizes, the ratio between the length of the non-fractal (straight) coastline of any island and the square root of its area is independent of the size of the island. However, for islands with fractal (irregular) coastlines the length $L(\delta)$ of the coastline depends on the yardstick δ used to measure its length, and $L \rightarrow \infty$, as $\delta \rightarrow 0$ (Feder, 1988). On the other hand, the area A (δ) of the island, measured by the covering it with squares of area δ^2, remains finite as $\delta \rightarrow 0$. Mandelbrot shows that for fractal (irregular) curves the divergent ratio _ should be replaced by the modified ratio given by

$$\rho_D = (\text{Perimeter})^{1/D} / (\text{Area})^{1/2} = [L(\delta)]^{1/D} [A(\delta)]^{-1/2} \qquad (1.2)$$

for each of the islands. The ratio ρ_D is independent of the size of the island-but it does depend on the size of the yardstick chosen. D is the fractal dimension of the indent coastlines of the similarly shaped island; it exceeds the topological dimension of the straight line (D=1) and stays somewhere in between 1<D<2. The concept of a distance between points in space is central to the definition of fractal dimension D. It derives from the number of boxes or another units of measurement needed to measure the distance and is now called the box counting dimension or box dimension (Feder,1988).

According to Mandelbrot, islands similar in form satisfy the following perimeter-area relation

$$L(\delta) = C\delta^{(1-D)}(A(\delta))^{D/2}, \qquad (2)$$

where $L(\delta)$ is the length of fractal line, which is measured by the “linear” length _ and, area by the squares with dimension $\delta \rightarrow 0$. $A(\delta)$ is the area of the island. D is the fractal dimension. C is constant of proportionality.

Mandelbrot (Mandelbrot, 1982) has pointed out that rivers satisfy the length-area relation in equation (2).

Interesting results on the Length-Area relation were obtained by Hack (1957). During his study of rivers in Virginia and Maryland, he received the following ration between the length of the longest river above a given location and its drainage area

$$L = CS\,D/2 \qquad (3)$$

where $1<D<2.5$.Where L is the length of the main river within the river basin; C is constant of proportionality; S is the drainage area.

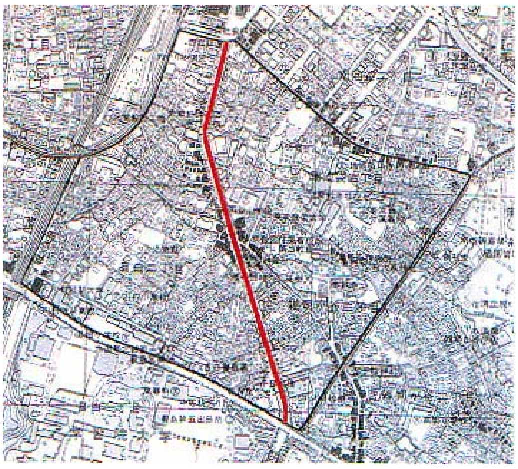
The logarithm of the Length -Area relation $L = CS^{D/2}$, reduces to the linear equation

$$y = (D/2)x + b \qquad (3.1)$$

where D=1.2 in the northeastern United States and D=1.4 in the western United States. Regardless of the geological or structural characteristics, Hack obtained the same numbers during an investigation of 400 rivers in the USA (Feder, 1988).

According to the Mandelbrot, fractals are shapes made of parts similar to the whole in some way. The configuration of the streets patterns within the limited territory resembles the fractal definition. During the research of the "chaotic" urban environment of Tokyo (1999-2000), we focused on defining the topological dependence between the sum of the streets within the limited "chaotic" zones (Shinjuku, Shibuya, Ikebukuro, Ueno, Nakano, and Aoto) and the area which they serve. In our research, we distinguished an order of the streets within the limited "chaotic" zones (Figure 1). The 1st street order proceeds from the main (wide) street until the first crossing, dead-end or branching, etc. The highest order _ , is assigned to the streets that have no tributaries. Along with street order, we further discerned the areas occupied by the streets within each order (Figure 2). As a result, we obtained the sequence of sums of the streets of all orders and the areas that they are occupied.

Figure 1: An example of a limited "chaotic" zone, Tokyo
(source: Topographical maps of Tokyo, 1993)



Our research discovered that the street zones do in fact have a fractal dimension – a result which fully concurs with Hack's investigations (Rodin and Rodina, 2000). Moreover, in our analysis, the fractal dimension in Tokyo's urban structure appeared within a higher order branching system. Hack's investigations exhibited fractal dimension in the lower order system. Surprisingly, all six chosen "chaotic" zones in Tokyo show the fractural dimension - the extent to which the fractal (in our case irregular streets) fills its available space(in our case limited territory by the main roads). This discovery proves that visual chaos of Tokyo's street patterns contains the hidden order that can be measured. In contrast to the irregular streets with the fractional (fractal) dimension, the standard regular grid (e.g. Manhattan) corresponds to the topological dimension of the area (surface) D=2 in well-known formula (2) of fractal dependence of the line from the area.

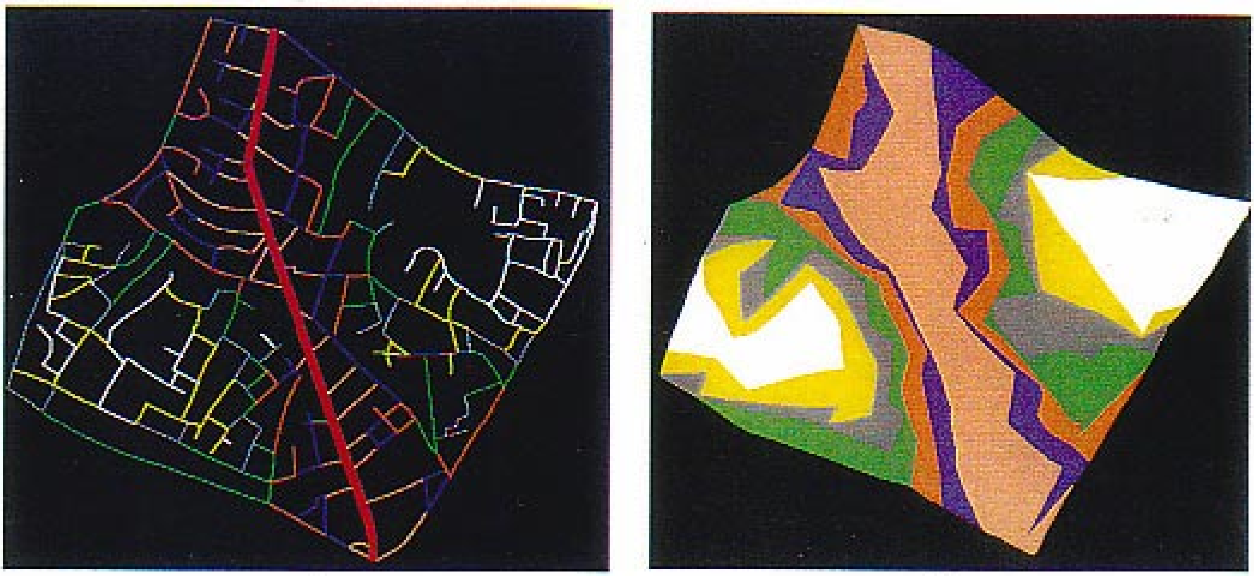

A B

Figure 2: A-street order classifications by color;
B- area order classifications by color
(created by the authors)

Further to our investigations of the fractal structure of Tokyo's streets, we created the test version of the computer program "Fractal". The program allows processing of the city map and defines the areas with a high index of fractal dimension. For dynamic graphic programming, the Sprite system was used. At the initial stage of the program's map of the city, we mark out by special color the Background (static part of the map): all dwellings, industrial enterprises, squares, parks, etc. All basic lines of communication (roads, highways, foot and bicycle routes) are uncolored. The dynamic Sprite then follows communication lines on the map, distinguishing the street order, and calculating the area occupied by the streets within each order.

To prove that the development of fractal geometry in this domain is not simply the indulgence of pure research, we apply the fractal analysis to the development of the strategic police deployment within the districts with the irregular street patterns. For many urban authorities round the world, irregular "feudal" street structure with the high index of fractal dimension is case of great concern. In most cities, the territories with the high irregularity in the street structure show a high rate of crime. Strategic planning of police distribution within urban areas has neglected the parameter of fractal dimension. We therefore propose that this neglect offers one explanation for the high percentage of crime concentration in these cities.

3. Development of the optimal methods and principles of police deployment within the "chaotic" urban structure

3.1 Background

In the 1970s, Richard Larson at MIT initiated the research on optimal methods of police deployment (Larson, 1978). Larson analyzed the advantages and disadvantages of the hazard formula – the most popular systematic method previously employed. This formula combines a number of factors into a hazard score that allegedly indicates the relative need for personnel in each precinct (Larson, 1978). Subsequently, Larson introduced a new method of police deployment- the hypercube queuing model. This model allows the police planner to consider quantitatively many details of a police precinct including: the different workloads of each neighborhood; and travel times from point to point. Larsen also promoted the use of the computer with simple interface.

Following Larson's research, a study was conducted in London by the leader of the Space Syntax, Bill Hiller, and his student, Simon Shu, on the criminal behavior. This study demonstrated the possibility of generating certain self-organizational rules of regulation (Hiller & Shu, 1999). Furthermore, in the mid 1990s, the Vera Institute of Justice, New York, developed the electronic crime mapping system, widely known as CompStart process. CompStart is a crucial tool for holding precinct commanders accountable for changes in the local crime rate, and for solving crime. In 1998, Vera launched a statewide system that could map crime across jurisdictions. However, Vera Institute members have not taken into consideration the index of irregularity of the city streets, while developing the optimal methods of the police deployment. With the discovery of estimating fractal dimension of street patterns comes a new approach to effective police deployment. This new non-linear analysis research was conducted in Russia.

3.2 The contemporary crime situation in Russia

The radical social and economic transformations taking place in Russia today have presented the community with both positive and negative phenomena. On the background of deficiencies in economy, there is a significant increase of the criminal activity. Intensive distribution received all kinds of property encroachments- particularly burglaries and aggravated thefts. The protection of the property of citizens is one of the main activities of the security police in Russia. With the object of activization of the work of the security police, police departments are trying to introduce the new results of scientific research into their work.

Analysis of the committed crimes from the protected objects (banks, shops, warehouses, and public organizations) shows that 45% of the thefts became possible due to the delay in the arrival of the operational police group on auto transport. The delay points to a significant and real ignorance of the shortest routs of movement within irregular street pattern. We consider the strategic planning of the optimal routs and places of concentration of the security police deployment to be a critical point in the reduction of the amount of burglaries. At present, the main patrol routs within the cities lie between the territory with the concentration of the valuable objects, and the territory with the irregular street pattern. It is safe to assume, that along the road with the low irregularity (e. g. strait grid) security patrol will reach any point on the protected territory quickly. Unfortunately, criminals intuitively choose districts with high fractal dimension for their activity. Within the fractal patterns, it is easier for criminals to escape from pursuit and it is more difficult for police to enter. The oversimplification of urban street planning during security police route development has resulted in an ineffective system.

3.3 The fractal optimization of the autopatrol routes of the security police (as in Sovetskii District of Voronezh City, Russia)

For our research we have chosen the example of the Sovetskii District of Voronezh city, Russia and put forward the following tasks:

1. To define the surface density of the valuable objects (banks, shops, warehouses, and public organizations) within the district, and to construct the map shows the surface of potentials.
2. By the method of least squares - to define a function, the graph of which will be the approximated surface of potentials.
3. To define coefficients of fractal dimension of the streets within the district and construct the map that shows fractal zones.
4. To receive a fractal approximated surface of valuable objects (by the multiplication of the corresponding elements of matrixes of the map valuable objects and of the discrete map of fractal zones).
5. To correct the patrol routes of the security police within the district by taking into consideration the maximums of the fractal approximated surface of the valuable objects.

Our research began by taking the situational map of the Sovetskii district of Voronezh (Figure 3) and, with the use of the obtained data on the valuable objects for protection, we plotted the co-ordinate 20X20 grid. The number of the valuable objects in each cell corresponds to the certain number of the cell in the grid (Figure 4). In 3D representation, it will look like a concentration of columns. To bring it closer to the real situation, we pulled the surface on the columns. The top view of the 3D representation gives us the map with the surface of levels of the valuable protected objects within the chosen district. The red color corresponds to the highest concentration of the objects (Figure 5). Obviously, the security patrol will be not able to drive up to each virtual column; and we are not able to carry out a patrol rout through the column. We then aligned the received data by the method of least squares. This method gave us the final map of the potential distribution of the valuable objects within the district (Figure 6). Afterwards, we calculated the optimal trajectory of the autopatrol movement within the district. There is a variational problem to also consider; extremes of which are the optimal routs of the security patrol movement (Dumachev, 2002). We found the results of the variational problem to be trivial, i.e.

the rout of the patrol movement is supposed to move perpendicular to the lines of the surface level in the direction of a gradient of potential. In this solution of the police deployment, time is not a critical parameter. Furthermore, there is no possibility to carry out the trajectory of movement through a maximum of the potential (sometimes the maximum of the potential lies in the living zone). Thus, the fractal dimension (dimension of indent irregularity pattern of streets in the district) directly connected with the interval of time the security patrol spent to reach different territories for inspection. Regarding the problem of optimization patrol routs, we therefore included the map of the fractal dimensions of the district as well as the map of the potentially valuable objects.

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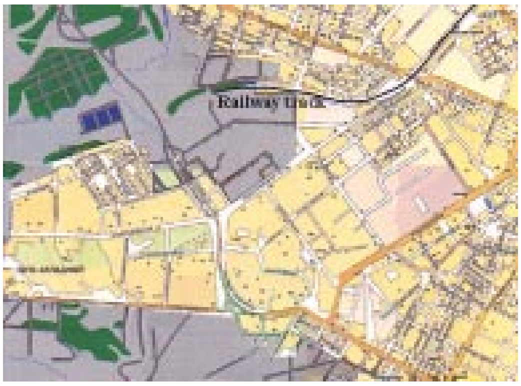
Figure 3: Map of the Sovetskii district, Voronezh
(source: http://map.vrn.ru)

Figure 4: Representations of the table of the valuable objects (created by the authors)



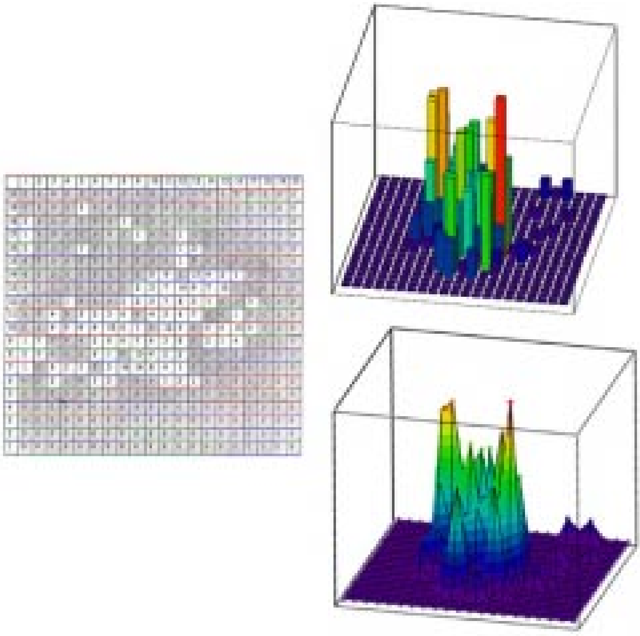
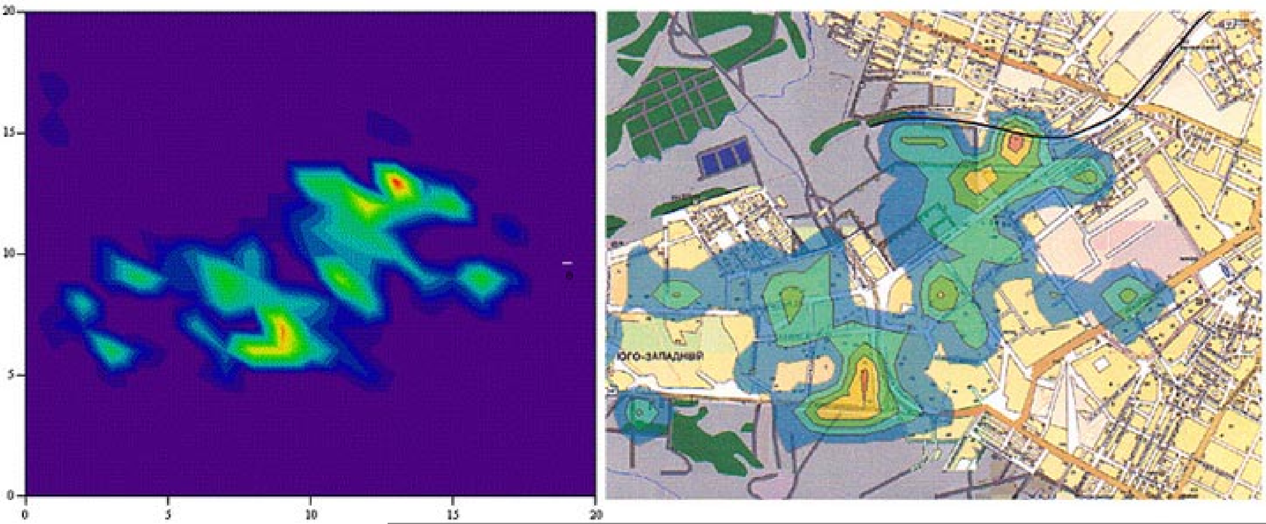


Figure 5: Map showing levels of the valuable object distribution (created by the authors)



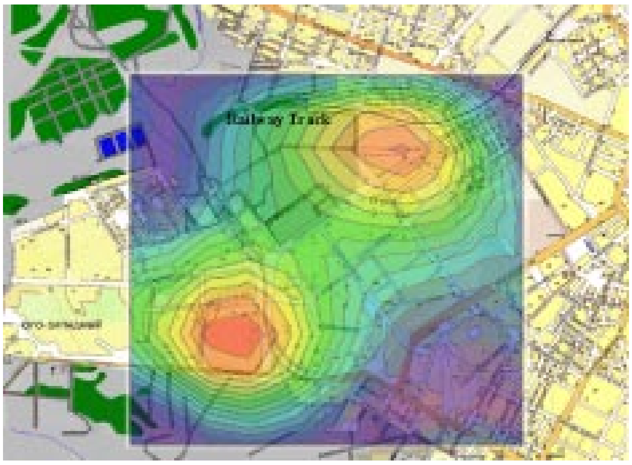


Figure 6: Aligned surface map of the valuable objects
(created by the authors)

For estimation of the fractal dimension of the street pattern in Voronezh, we used the test version of the computer program "Fractal". In fact, we received the conditional fractal map (Figure 7). On the obtained map, we plotted the co-ordinate 20X20 grid. The numbers of the fractal dimension (correlation of logarithms of Area-Length relation of different street order) placed into the table indicates the numbers of fractal dimensions (dimension of complication) of each "fractal belts". We then aligned the received data by the method of least squares. This operation gave us the final map of the fractal dimension of the streets (indexes of irregularity) within the district (Figure 8). Finally, we multiplied the corresponding cells of the table of the valuable objects distribution and the table of the fractal dimension of the streets patterns. On the basis of the data received, we created the discrete map that shows the surface of potentials and take into account the fractal dimension of the streets, and where the valuable objects located (Figure 9).

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
20	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
19	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
18	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
17	1	1	1	1	1	1	1	1,22	1,22	1	1	1	1	1	1	1	1	1	1	1
16	1	1	1	1	1	1	1	1,24	1,3	1,22	1,22	1	1	1	1	1		1	1	1
15	1	1	1	1	1	1	1	1	1	1,24	1,3	1,3	1,26	1,22	1	1	1	1	1	1
14	1	1	1	1	1	1	1	1	1	1	1	1	1,24	1,26	1,22	1	1	1	1	1
13	1	1	1	1	1	1	1	1	1,36	1,16	1	1	1	1,2	1,23	1,13	1	1	1	1
12	1	1	1	1	1	1,36	1	1	1	1,16	1,1	1,1	1,18	1,23	1,2	1,13	1	1	1	1
11	1	1	1,15	1,15	1,26	1,36	1,15	1	1,36	1,1	1,1	1,15	1,22	1,22	1,13	1,13	1,1	1	1	1
10	1	1	1,15	1,15	1,15	1,15	1,15	1,13	1	1,13	1,13	1,22	1,23	1,18	1,13	1,13	1,1	1	1	1
9	1	1	1,13	1,1	1,07	1,07	1,07	1,07	1,05	1,13	1,2	1,22	1,18	1,13	1,07	1	1	1	1	1
8	1	1	1,13	1,13	1,13	1,1	1,13	1,13	1,05	1,13	1,2	1,2	1,13	1	1	1	1	1	1	1
7	1	1	1	1	1	1	1,05	1,05	1,05	1,05	1,07	1,07	1,05	1	1	1	1	1	1	1
6	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
5	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
4	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

Figure 7: Levels of the fractal dimension
(created by the authors)

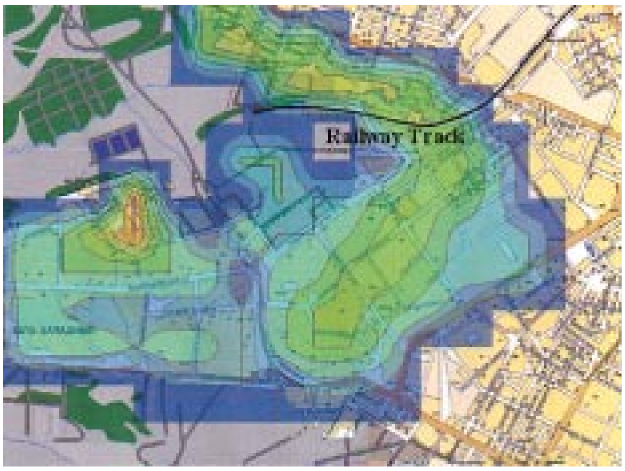


Figure 8: Map showing aligned levels of the fractal dimension
(created by the authors)

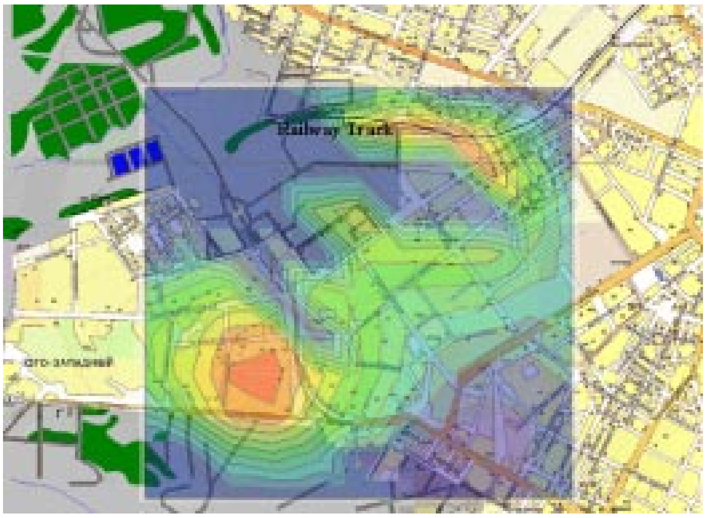


Figure 9: Final map showing the optimized zones of police deployment
(created by the authors)

If we compare the matrix of valuable objects and the final map, we can observe that in the upper right corner of the map, the red zone shifted by bend along the railway track. This is very interesting result. Along the railway track lie a number of private houses and a very complex fractal street pattern. There are no valuable protected objects in this district so it follows, there is no patrol. However, according to the statistics, it is precisely this district criminals disappear into after the robbery of the valuable objects on more regular street pattern. The results of our research became a recommendation to the local police department to pay special attention to the zone along the railway track).

4. Conclusion

We hope that our results will help to build the new theoretical and practical foundation for approaching city analysis and strategic planning. We believe that our research will also become one of the components of the global research on the structure of the natural systems to which the city refers. We hope that our research results will help us to collaboration with Space Syntax organization, whose activity we are very interested in.

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