

## GEOMETRY AND SPACE IN THE ARCHITECTURE OF LE CORBUSIER AND MARIO BOTTA

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### 0 Abstract

Architects construct and represent buildings in drawings using geometrical rules. Drawings embrace geometry as a totality that is exposed to vision at once. Space embodies geometry as a totality that is revealed sequentially through movement. This paper looks at the ways in which the geometrical rules that organise buildings in drawings are revealed and grasped when moving in space. This is examined in the context of four villas of Le Corbusier and four houses of Mario Botta. A top-down analytical approach is proposed that progresses in stages from abstract geometrical concepts to a detailed description of a building. It looks at the ways in which invariant geometrical properties and reversible operations establish interconnections amongst higher and lower states of abstraction. Analysis reveals that in Botta there are global scale geometrical characteristics that remain invariant in the transformation of visual fields produced by movement. In Le Corbusier there are only local scale invariant characteristics picked up during movement in space. In Botta, geometry in drawing and geometry in space are coordinated guiding intelligibility towards a single reading. In Le Corbusier they develop in different directions accommodating multiple interpretations.

It is argued that the two kinds of intelligibility are based on two different compositional logics. In the first one composition is dominated by pre-existing geometrical principles from the early design stages, (Botta). In the second one pre-conceived principles interact with combinatorial possibilities emerging during a design process, (Le Corbusier). The former generates buildings that are grasped at once subjecting spatial narrative to geometrical pattern. The latter results in buildings that demand intense attention and extensive exploration making spatial progression the main protagonist of spatial experience. This paper concludes that a top-down approach seems relevant to design operations. Designs are generated by abstract geometrical definitions which are gradually transformed into detailed drawings. This approach studies geometrical systems as transformations from abstract to detailed stages producing in reversible order a shaping logic of composition. In this way, it is proposed that a description of geometry and of its embodiment in space must look at composition as a dynamic activity in process.

### 1 Introduction

There are two ways of walking inside a building. The first one is to try out a route that serves a specific purpose, like seeing a display in a museum. The second one is to walk as to discover what the building is like and how the architect has sculptured it through a system of geometrical rules. Some buildings in addition to social purposes arouse interest and stimulate a viewer to take the second kind of route. Viewers might perceive buildings in various ways based on their own systems of understanding.

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Keywords: *architecture, composition, drawing, geometry, movement*

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However, in spite of their private perspectives, what they experience from within has been conceived from without and orchestrated on a drawing board through a system of rules. It is this orchestration a viewer might feel stimulated to uncover. This is also the question which this paper tries to answer:

*How are the geometrical rules that shape a building in a drawing seen and grasped when moving in space?* This question is investigated in the context of four villas by Le Corbusier and four houses by Mario Botta (illustration 1).

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### 1.1 Geometry and its embodiment in drawings and space

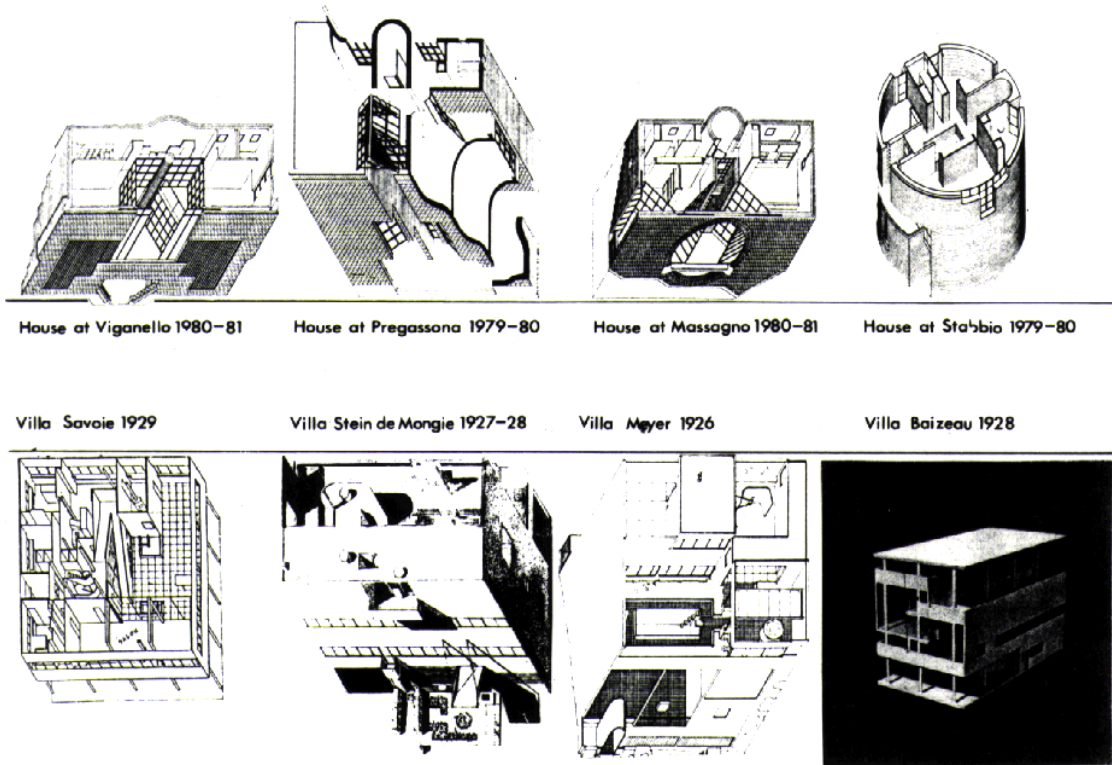


Illustration 1.

To explain the ways geometry is seen in drawings and the ways it is seen in space, one can start with elementary layouts, (figures 1 and 2). In a drawing, these layouts are seen as two spatial rectangles, the geometrical centres of which are covered by the same axis. In space they are seen separately as two spatial enclosures separated by a surface, (figures 3, 4 and 5). Therefore, in drawings, layouts are visible at once. In space, they are seen through a set of visual fields that are constrained within what is accessible to the eye. In drawings, visual information is always the same. In space, it constantly changes as one moves from one place to the other. In drawings, seeing aided by the static characteristics of the visual field is straightforward. In space, it is integrated within the complex experience of movement and is sequential. There is usually no point which reveals the relation between the parts that are encountered sequentially and the whole.

Thus, geometry in a drawing co-ordinates and synchronises spatial locations on a flat surface. It is intrinsic to the notion of space but not identical to the space it represents. An examination of ways geometry is experienced in the spatial domain must proceed from an analysis of the system of representation, to the thing that is represented, i.e. from drawings to space.

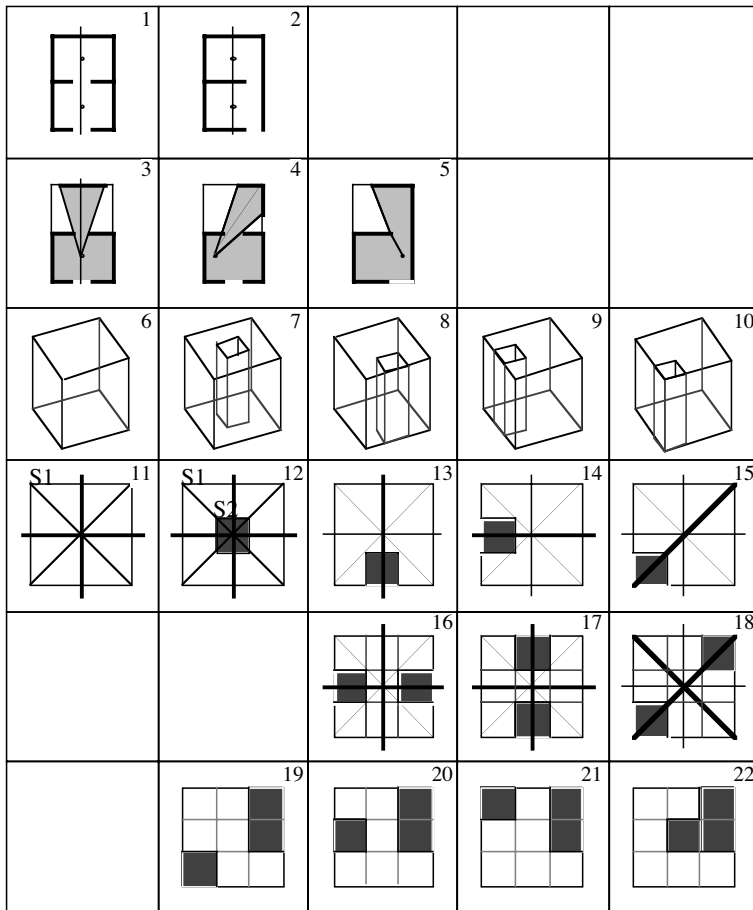


Figure 1

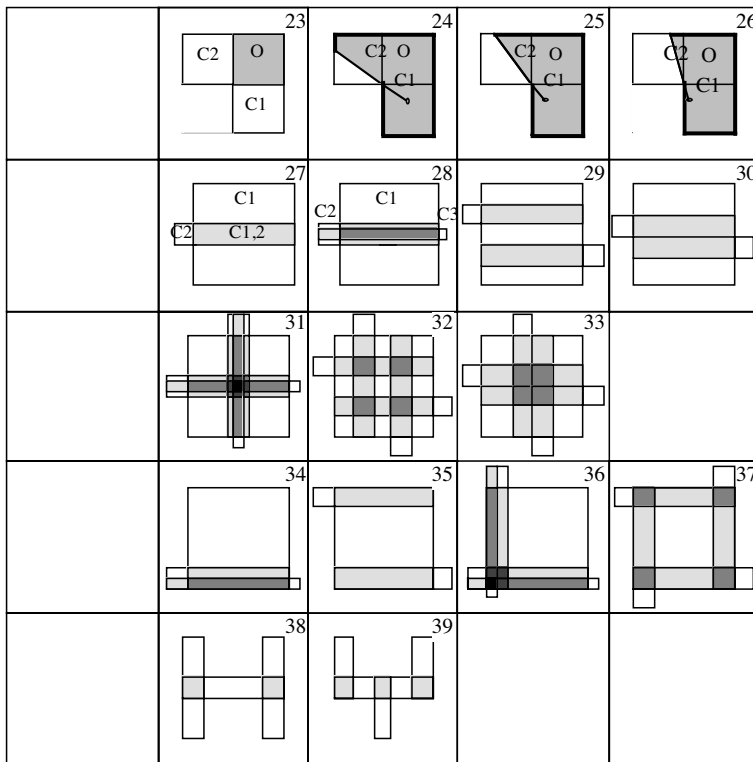


Figure 2

**2 The Analytical Method**

As opposed to analytical methods which dissect a system into its small scale components and study their relationships, a top-down analysis is proposed instead. This progresses in stages from the volume to the plan, from the large to the small scale and from the most abstract geometrical concept that describes a building down to its

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detailed description. A series of analytic stages are defined which are examined separately as well as in relation to the others. In this way, lower and higher levels of abstraction are associated establishing a description that progresses from an abstract to a specific state as well as in a reverse order.

### 2.1 *Geometry in drawings*

#### 2.1.1 *Describing geometrical properties as transformations*

The analytical method can be explained by looking at figures 6-10. These are seen as resulting from the subtraction of a volumetric rectangle from the centre (figure 7), from the front (figure 8), from the left side (figure 9), and from the corner (figure 10), of a cube. Properties are examined in two directions by looking at horizontal sections cutting through the volumes (figures 11-15). Figures 11 and 12 have four reflective symmetries. Figures 13, 14 and 15 retain symmetry on the vertical (figure 13), the horizontal (figure 14) and the diagonal axis (figure 15). In these configurations a simple geometrical concept (S1), is transformed to more complex ones (S1-S2), in a way that some of its symmetries are broken while others are retained. *Thus, the top-down method describes properties as transformations in which higher and lower levels of abstraction (S1, S1-S2), are linked together by an operation (subtraction), and geometrical rules that remain invariant.*

#### 2.1.2 *Shape, grid and physical properties*

This method looks at invariant geometrical rules at three layers of properties: *shape*, *grid* and *physical properties*. These can be explained by looking at figures 16-18. In these figures two volumetric rectangles are subtracted from the cube. The lines defining the small rectangles are extended, subdividing the larger one into nine square units. Two different views of these configurations can be taken into consideration: one is an arrangement of square grids - the other is a combination of shapes (S1, S2). The grid arrangement retains all reflective symmetries of S1. The shape arrangement in figures 16 and 17 has two reflective symmetries on the vertical and the horizontal axis. In figure 18 it has a single reflective symmetry on the diagonal axis. Therefore, whereas the former retains all kinds of symmetries, the latter reduces their number. Thus, *shape* and *grid properties* can take a similar or a different pathway in a transformation.

The different directions these properties can take is explored further by comparing figures 16-18, with figures 19-22. In both sets of figures the grid symmetries are preserved. However, whereas figures 16-18 preserve certain symmetries of the largest shape, figures 19-22 break all these symmetries. In figures 16-18 shape and grid symmetries are co-ordinated at the level of two reflective symmetries. In figures 19-22 there is no co-ordination between the two layers.

The distinction between shape and grid properties enables a study of approximate symmetry that emerges through the relations amongst extended lines. It also enables a distinction between *explicit* and *implicit* geometrical patterns. The former are explicitly represented by shapes and their contours. The latter are implicitly present in the form of lines forming the extensions to these contours. The importance of physical elements in carrying a structural pattern to the observable level is explored further in the section that follows.

### 2.1.3 *Physical properties*

In figures 13, 14 subtraction breaks the continuity of the lines defining the large and the small rectangle. In figure 15 it breaks the continuity of these lines, also removing a vertex. In the former, the rectangle is physically recognisable, retaining the reflective symmetries of its four vertexes. In the latter, the symmetries of the front left vertex are broken, resulting in a new shape. Therefore, there is a third layer of properties which is carried by the vertexes of a shape. These are called *physical properties* since they seem to give physical definition and recognisability to a shape.

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In figures 12, 13, 14, 16 and 17 all layers of properties are co-ordinated retaining certain symmetries invariant. In figures 15, 18 it is only the shape and the grid properties that are co-ordinated, retaining reflective symmetry on the diagonal axis. In the former, S1-S2, S1 and S2 are all recognisable as a firm system. In the latter, the recognisability of S1 and S2 is sustained by the shape and grid symmetries only. The removal of a vertex decomposes this shape, producing a tension amongst the three layers.

Thus, the analytic method approaches geometrical order as a structure organising relations amongst shape, grid and physical properties. When these layers are co-ordinated by the same rules they seem to operate as a single system. When they are governed by different rules they function as different systems. In the first case there seems to be a single interpretation constructed by a common rule. In the second one there is more than one interpretation leading to different directions which are in tension with each other.

### 2.2 *Geometrical properties in space*

To explore the ways geometry is seen in space one may begin with figures 1 and 2. Isovists are used to describe visual experience in these layouts (Benedict, 1979). These are visual fields consisting of the physical elements that are visible from a specific point and of the radials connecting these points with the edges of surfaces. In figure 3, the geometrical centres of the triangles defining the visual field of an observer standing in front of the door, lie on the axis that connects the geometrical centres of the rooms. Every field constructed along this axis retains this principle invariant. In contrast, the axes of the fields constructed from every other point change (figure 4). *There is a geometrical pattern in visual information that remains invariant as a person moves in space.* In figure 5 every visual field is defined by the surface that connects the two spaces. In this case the geometrical property that stays invariant is the binding surface.

The effects of invariance in visual information have been outlined by Gibson who argued that perception does not depend on a series of still images that are accumulated through time and stored for comparison, but on invariant information picked up through motion (Gibson 1952). Influenced by Gibson, Hagen argues that the transformation in the information provided by pictorial and retinal images is geometrically determined (Hagen 1985). An observer moving in relation to a regular solid sees congruent images from different sides. Moving towards a surface along a frontal axis one sees an expansion of this surface. Its size changes but its shape remains the same. In the case of a perspective image both the size and the shape of the surface change. However, its edges project the same cross ratio from different view

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points so that those edges are seen as straight and the surface as flat.

Therefore, geometrical invariants are informative in the process of understanding geometry in a layout. Based on this observation it could be argued that in figures 1 and 2 a viewer can synthesise the partial views into a geometrical schema through the invariant property of the axis to organise a series of fields (figure 3), or the invariant property of a surface to define these fields (figure 5). *It seems that the ways in which geometry is grasped during spatial experience can be examined by looking at the invariant geometrical characteristics in visual information.*

### 2.2.1 *Overlapping convexity - centralised and dispersed visual information*

The problem then becomes one of defining an analytical tool that describes invariant geometrical information picked up as one moves in space. This is possible by the *overlap convex space modelling*. This is a technique that draws convex spaces on a plan and looks at their interconnections. A convex space is defined by Hillier and Hanson as the area in which every line connecting two of its points goes inside this space (Hillier & Hanson, 1984). In figure 23 any point in the area defined by the extensions of the lines that constitute the vertex (O), reveals the whole space at once. From the regions C1 and C2, information constantly changes (figures 24, 25, and 26). From this area it stays the same covering the actual shape of the layout. This is because two convex spaces (C1 and C2), overlap at O which provides information about the arrangement as a whole.

The overlap space modelling is examined further in figures 27-39. In figure 27, a convex space, C2 intersects with C1, resulting in an overlap unit (C1, 2). In figures 29 and 30 two convex spaces overlap with C1 constructing two overlap units (C1, 2, C1, 3). In figure 28 C2 and C3 overlap with C1 as well as with each other. A different shade is assigned to the overlap units according to the number of spaces to which they belong. The dark-light distinction indicates a distinction between units belonging to a large number of convex spaces and units belonging to fewer ones.

In figure 28 all three spaces can be seen simultaneously from C1, 2, 3. In figure 29 one has to move from C1, 2 to C1, 3 in order to see the spatial configuration as a whole. In figure 30 one sees all three spaces by moving along the common line of C1, 2 and C1, 3. Therefore, simultaneous information is also possible from overlapping units that share their defining sides.

In figures 31-37 convex spaces are added to the initial space overlapping with the existing ones along the other direction. In figures 28, 31 and 33 global scale information is transmitted from the centre of the layout. Figure 32 exposes information according to a chequerboard pattern of overlap units requiring one to move from one dark unit to the other. In figure 34, the layout is seen as a whole from the area situated next to one side of S1. In figures 35-37, one has to move from one side of C1 to the other to see the small spaces attached to the main one. A more dispersed pattern of visual information is presented in figures 38 and 39. In these layouts the linear extension of space together with the dispersed distribution of overlap units intensifies movement. In this way, two kinds of visual exposure are identified. One is a simultaneous exposure from few points. The other one is a successive exposure from many points.

However, regardless of the degree of movement required to see these layouts as wholes, the arrangement of shades reveals the geometry of the plan. Thus, the distribution of information follows the rhythmical spacing of the grid bays exposing the interconnection of these spaces, of their surfaces and their grid lines. In these layouts, geometrical properties are exposed in spatial experience.

Therefore, as opposed to isovists that capture experience as a collection of changing visual fields, overlapping convexity describes experience as a structure of fields. It is characterised by descriptive economy overcoming the need to draw limitless isovists from all spatial points in a space. Another crucial dimension of this method is that it captures the exposure of surfaces defining convex spaces and the underlying network of grid lines that establish interconnections amongst these surfaces. In this way, a single analytical tool describes the visibility field, its physical definition by boundary walls and their geometrical structure. Besides, overlapping convexity carries invariant geometrical characteristics into the level of analytic representation and into the level of spatial experience. As such, it seems to enable a study of the relationship between geometry seen in space and geometry seen in a drawing.

Finally, the overlap space modelling brings design elements, like shapes, surfaces and grid lines, into the realm of analytic representation. Therefore, it constructs an homology between tools used in analysis and those used in design. The implication of this homology is a study that progresses not only from geometry seen in space to geometry seen in drawing but also in a reverse order. This is because it reveals the ways in which certain design decisions constructing overlapping shapes and intersecting lines on a plan create certain kinds of geometrical exposure in spatial experience.

### 3 Analysis of houses by Mario Botta and Le Corbusier

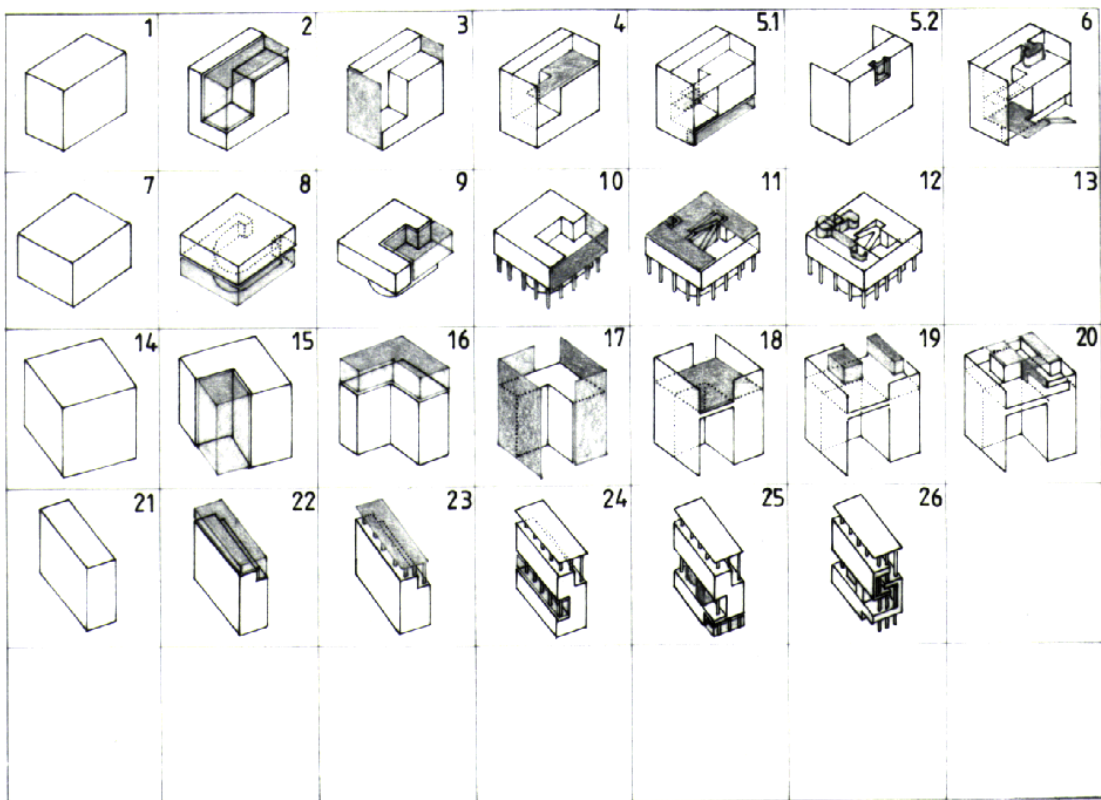
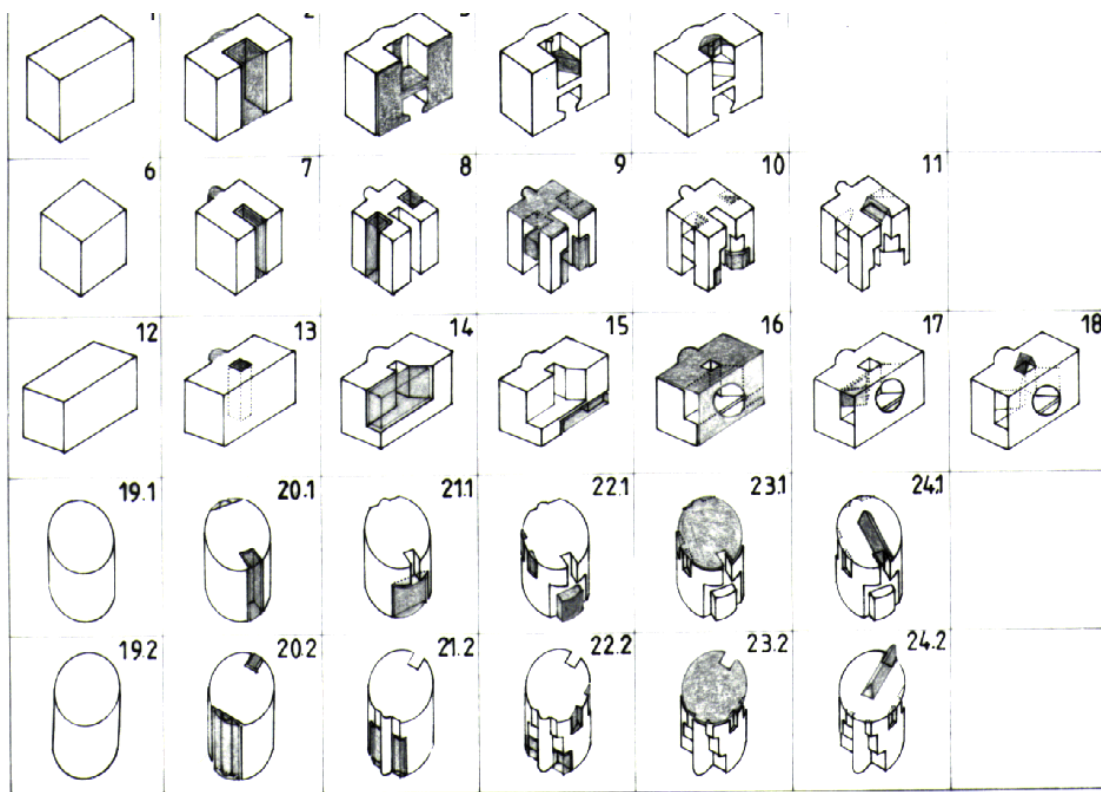
With this analytical framework and tools, the description moves to the selected buildings. The aim is to see what these architects do in their geometrical articulation of the volume and the plan and how their geometrical strategies are revealed to an observer moving inside their houses.

The top-down analysis starts with the largest geometrical solid that describes the overall form of the building and gradually dissects this volume moving to increasing levels of detailing of its external appearance (volumetric analysis, see Tables 1, 2). Then it progresses to the plan gradually introducing lower levels of internal articulation (plan analysis, see Tables 9, 10). Finally, it moves to the ways geometry is seen in space, looking at how the higher and the lower levels of the internal articulation established in the analysis of the plan, are experienced from the inside (spatial analysis, see Tables 11, 12, 13).

#### 3.1 Volumetric Analysis

The analysis of the volumetric appearance of these houses will show that in Botta there is a systematic preservation and co-ordination of the physical, the shape and the grid properties of the largest geometrical solid. In Le Corbusier there is a preservation and suspension of these properties. Botta constructs a geometrical unity and a single reading that is available at once from a privileged frontal view point. Le Corbusier creates a geometrical complexity that branches out in multiple readings, inducing the viewer to shift positions and engage in extensive exploration.

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Tables 1 and 2. Top-down analysis of the physical articulation of the volumes (Mario Botta, Le Corbusier)



### 3.1.1 Describing the transformation - operations and their order of occurrence

The simplest geometrical concept that describes all buildings is a prime solid - a block (Table 1, figures 1, 6, 12, 19.1, 19.2; Table 2, figures 1, 7, 14, 21). This is transformed by common operations - subtraction, addition and planar extension. Transformation starts with the subtraction of a volumetric component from the block (Table 1, figures 2, 7, 13, 20.1, 20.2; Table 2, figures 2, 8, 25, 22). This operation occurs at the next stages, excavating the house further, (Table 1, figures 8-10, 14, 15, 20.1, 21.1, 22.1, 20.2, 21.2, 22.2; Table 2, figures 5.1, 5.2, 9, 16, 24, 25). It is combined with the extension of horizontal and/or vertical planes and with addition. Extension restores some of the missing surfaces and redefines the largest solid (Table 1, figures 3, 9, 16, 23.1, 23.2; Table 2, figures 3, 10, 11, 17). Addition attaches geometrical solids or planar elements, subdividing the open space into subspaces (Table 1, figures 3, 4, 9, 10, 17, 22.1; Table 2, figures 5.1, 25). At the last stages, addition introduces volumetric elements at the top of the buildings (Table 1, figures 5, 11, 18, 24.1, 24.2; Table 2, figures 6, 12, 19).

It seems that there is a consistent pattern of transformation that employs specific operations and associates them with certain stages in both architects. However, the shapes used, their combinations and ways in which the volumes are transformed, are different in one category of buildings from the other.

### 3.1.2 Shapes and combinations

At the first stages, Botta always subtracts a rectangular volume (Table 1, figures 2, 7, 13, 20.1, 20.2). Le Corbusier in two houses, subtracts a volumetric L (Table 2, figures 2, 22), whereas in the others he subtracts a curvilinear volume and a volumetric rectangle (Table 2, figures 8, 15). Botta hollows out the block always at the centre. He creates a volumetric U which opens always towards the front. Le Corbusier varies the position of the subtracted component creating voids that open at least towards three sides. Botta is consistent regarding the shape of the subtracted element, its position and the shape of the resulting solid. Le Corbusier uses different shapes, varies their positions and uses subtraction to create different solids.

Therefore, Botta has a fixed repertory of simple shapes and combinations. Le Corbusier's repertory is complex, larger and flexible. However, regardless of their differences in physical appearance, Le Corbusier's resulting solids are all volumetric Ls in one or in three directions. A three directional L is a volumetric L that provides with horizontal, vertical across and vertical along sections that are all planar Ls like the volume in figure 2, Table 2. An one directional L gives sections that are planar Ls along a single direction (Table 2, figures 15, 22). Therefore, although there is no recurrent shape, there is a recurrent type of shape. Although there is not a recurrent position of the subtracted element, there is a recurrent type of combination requiring the volume to open towards more than one side.

A recurrent repertory of simple shapes and combinations in Botta results in houses that look similar to each other. Le Corbusier's repertory of complex shapes that differ in terms of physical appearance and position creates a difficulty in isolating shapes and classifying them into clear categories. This seems to generate houses that look different from each other. The effects shapes and their combinations have in the

transformation are examined in the following section.

### 3.1.3 *The rules of the transformation process*

#### 3.1.3.1 *Physical properties*

By excavating from the centre, Botta retains the physical definition of all the vertices of the block. Le Corbusier opens up the block at the corners, destroying its physical definition. Botta's volumetric U retains the physical properties of the initial solid. On the other hand, Le Corbusier's L suspends these properties. In the former, the block is physically present. In the latter, it can be retrieved only by the extension of the missing elements and is, thus, implicitly present.

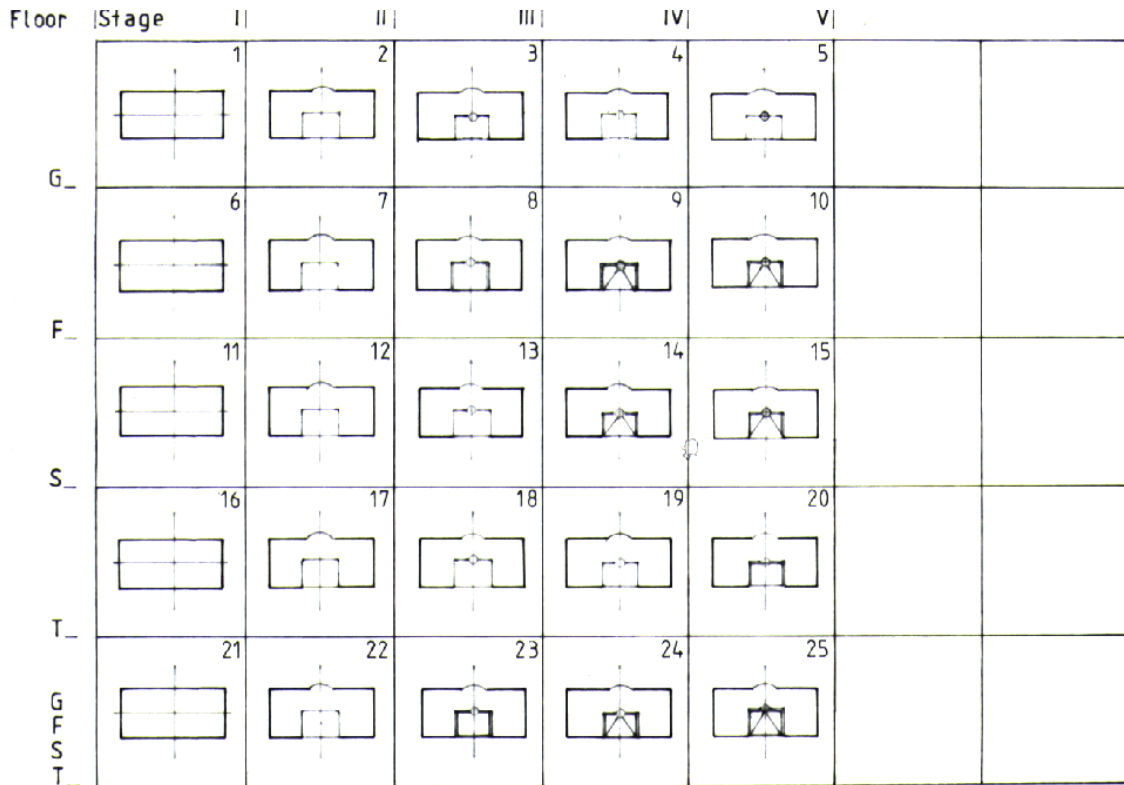
In Botta subtraction and addition do not affect the vertices and the edges of the initial volume at any stage. Planar extension is an additional device that increases the physical demarcation of the initial solid. In Le Corbusier, there are stages in which the physical definition of the block is reduced further through the subtraction of smaller components. There are also stages in which planar extension redefines this volume. Therefore, in Botta, the physical properties of the block restrict the transformation process throughout the stages. In Le Corbusier, these properties are alternatively suspended and preserved.

Nevertheless, Le Corbusier's extended surfaces do not fully define the missing vertices, (Table 2, figures 3, 10, 17). They register as planes decomposing the block and the volumes resulting from subtraction to an interaction of planar and volumetric components. Botta establishes the hierarchical distinction of the block from the rest of the elements and makes it clearly distinguishable as a physical concept. This is achieved by a clear distinction between its surfaces and those of the smaller components (excavation at the centre). Changes in the latter do not affect the former. Le Corbusier creates a mutual interconnection of elements that provides conflicting readings. This is based on a definition of the block and of the rest of the volume by the same surfaces (excavation at the corner). The decomposition of the former defines the latter. The re-definition of the latter decomposes the former.

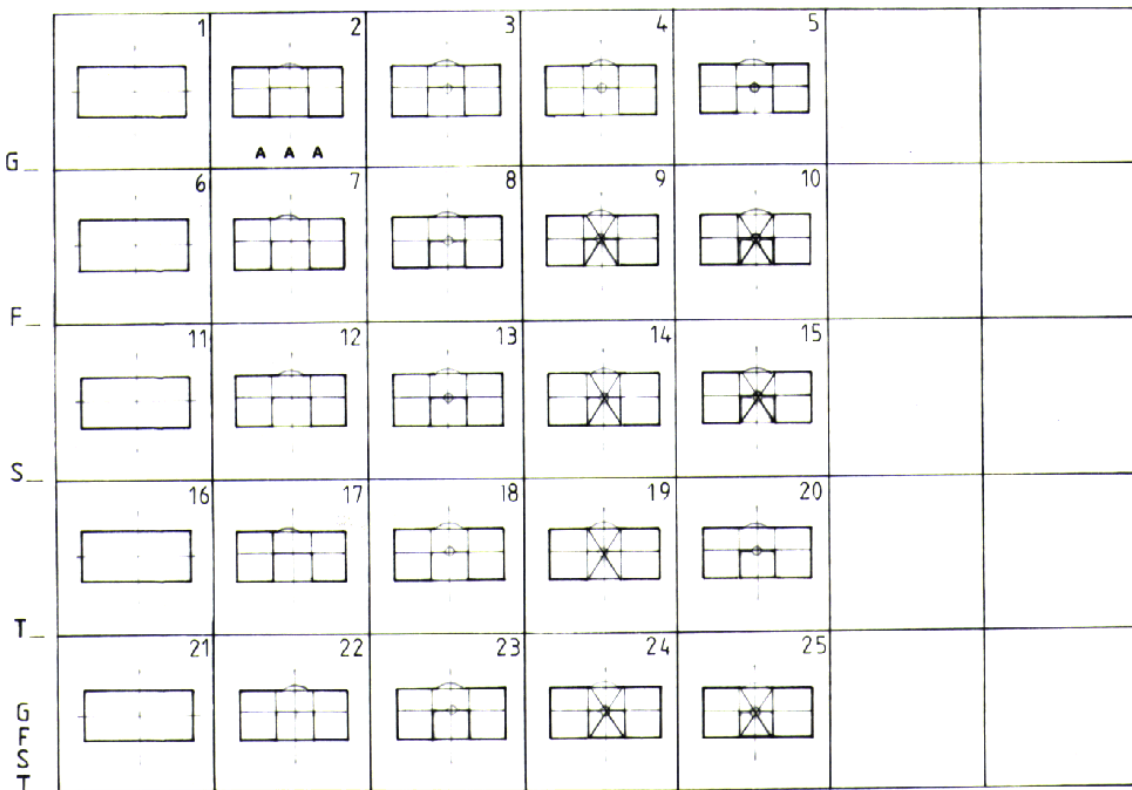
Gestalt psychologists suggest that shapes that share their boundaries generate ambiguous figure and ground relations. Each time perception focuses on one element the other fades into the background (Arnheim, 1974). This analysis proposes that ambiguous readings are created not by perception shifting its focus from one object or the other, but by a number of elements, properties and reversible operations. Figures 3, 10, 17 and 23 are ambiguous because both subtraction and extension are incorporated pointing at opposite directions: one towards the definition of the voids and the solids resulting from subtraction, the other towards the re-definition of the block.

#### 3.1.3.2 *Shape and grid properties.*

Shape and grid properties are examined by looking at horizontal sections through different floor levels and at the roof plan (Tables 3-8). A superimposition of these sections enables a study of properties at the level of the volume as a whole (see Table 3, figures 21-15; Table 5, figures 25-30). This is to enable an examination of buildings that change along the third direction like Le Corbusier's villas. This part of the analysis is illustrated by one house of Botta and two villas of Le Corbusier. However, the



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Tables 5 and 6. Shape and Grid properties in horizontal sections cutting through the volume (Mario Botta, house at Viganello)

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properties discussed here were found to operate across the whole sample.

In each horizontal section in Botta, the subtracted and the added shapes are symmetrical on the back to front axis covering the geometrical centre of the block (Table 3, figures 1-20). The grid lines defining these shapes are symmetrical on this axis also (Table 4, figures 1-20). Therefore, the back to front axis organises symmetrical relations amongst shapes and grids at the level of the volume as a whole. Similar to the physical properties, the shape and the grid properties of the block direct the volume in process throughout the stages.

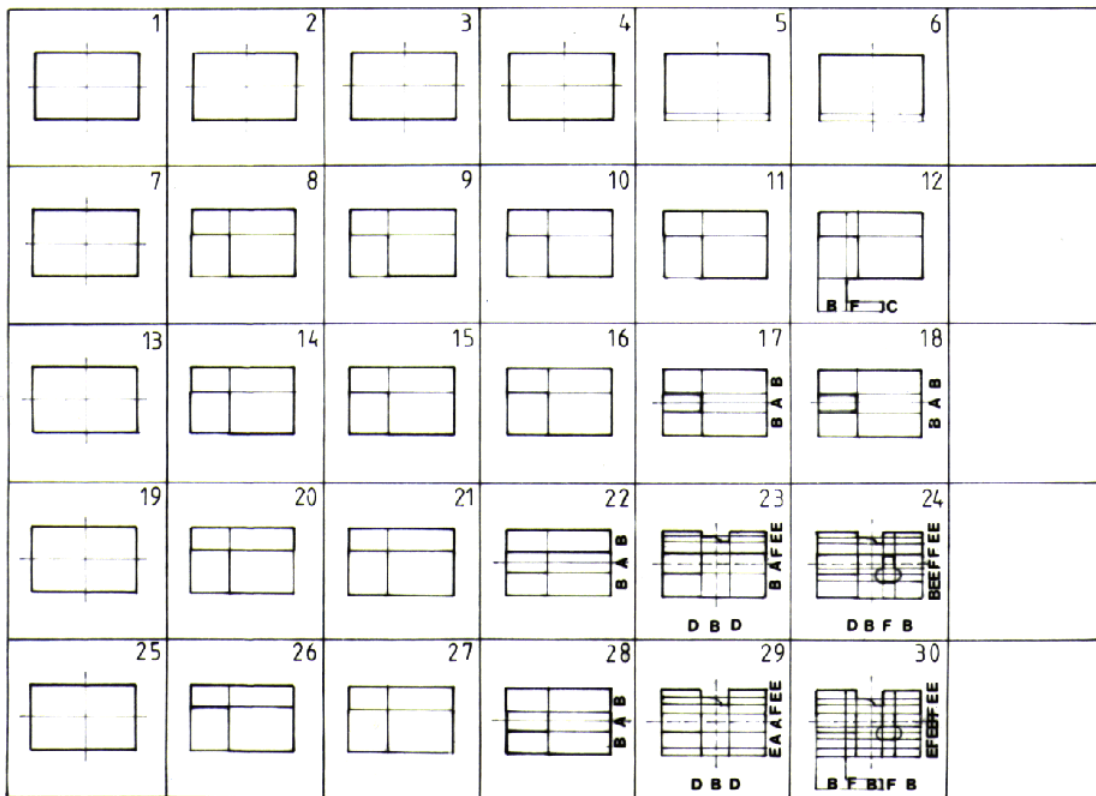
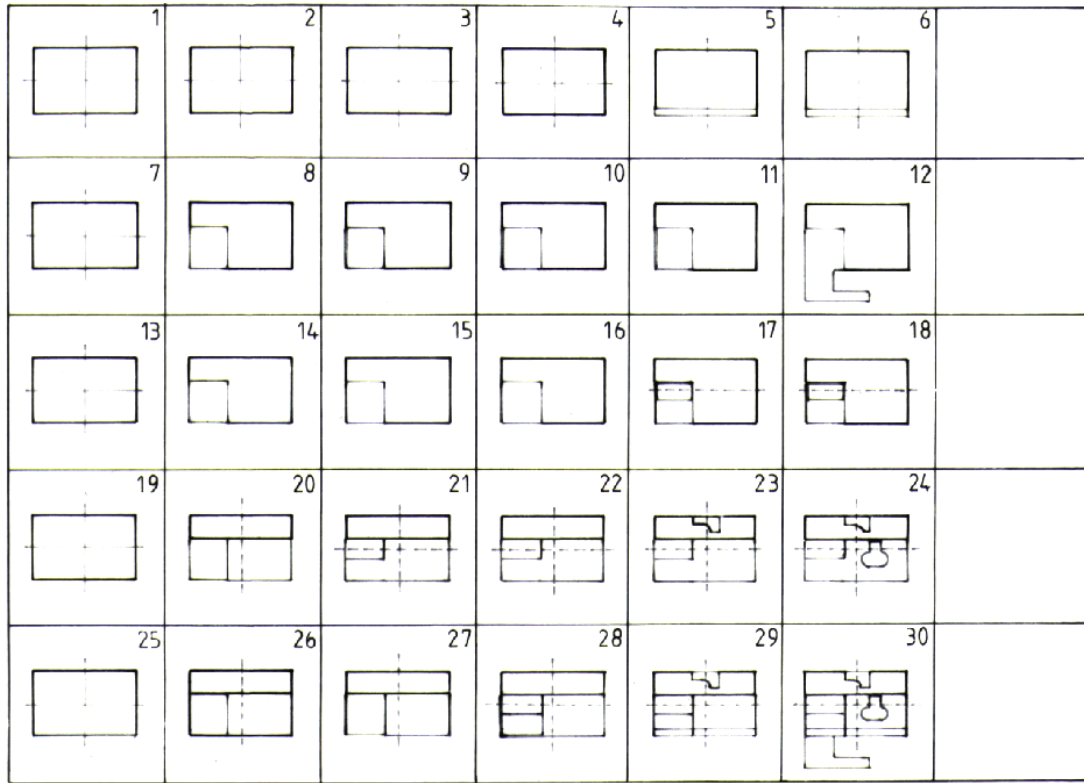
In Le Corbusier the organisation of shapes moves from symmetry at the first stages (Table 4, figures 1, 2, 7, 13 and 19), to asymmetry at the last ones. However, even at the first stages, symmetry is not a three dimensional principle. A contrast is created between the organisation at the level of the volume as a whole and the organisation of individual levels. This can be seen by looking at the overall asymmetry of Villa Stein at stage two, observed in the superimposition of all horizontal sections (Table 5, figure 26), and the symmetrical organisation of the ground floor (Table 5, figure 2). Another example is given by the overall symmetry of Villa Savoie along the back to front axis (Table 7, figure 20), and the symmetry of the first floor level and the roof plan (Table 7, figures 8, 14), on both back to front and left to right axes.

The arrangement of the grids in the superimposition of sections is symmetrical on the back to front axis (Table 6, figure 30; Table 8, figures 23, 24). This is introduced by the small scale articulation occurring at the final stages. Nevertheless, overall symmetry is contrasted either by asymmetry of a horizontal section (Table 6, figure 12, 24; Table 8, figure 18), or by symmetry on the left to right axis (Table 6, figure 18). Therefore, there is a dissociation between the organisation of the volume as a whole and the organisation of the individual levels at both, shape and grid properties.

A dissociation is constructed between the shape and the grid organisation also. This is based on a contrast between asymmetry of the shape arrangement (Table 5, figure 30; Table 7, figure 24), and symmetry of the grids (Table 6, figure 30; Table 7, figure 24). Thus, the explicit and directly observable symmetries of the shapes are questioned and replaced by implicit symmetries of regulating lines. Nevertheless, regardless of the implicit order of the grids, the differences in the grid properties amongst individual levels, disturb a three dimensional integration into a single organising principle. This can be seen in the symmetrical grid of the superimposition of horizontal sections in Villa Stein, where the lines of the elliptical volume at the third floor are combined with the lines of the first floor terrace to define a B F B F B rhythm (Table 6, figures 12, 24 and 30). Looking at the volume, this rhythm is not evident at a single glance (Table 2, figure 6). This is because it is disrupted by the difference in the physical appearance of the elements it springs from. It is also because it is generated by elements that belong to different horizontal levels.

To summarise, Botta co-ordinates all layers of properties by a single rule that is hierarchically applied from the largest to the smallest element. Le Corbusier dissociates these layers, preserving the principles of the largest element at one layer, while suspending them at another layer. The simultaneous application and suspension of prop-

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Tables 5 and 6. Shape and Grid properties in horizontal sections cutting through the volume (Le Corbusier, Villa Stein)

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erties is also applied within each layer. Thus, there is a decomposition and a redefinition of vertices. There is shape and grid symmetry at the level of the volume as a whole, and shape and grid asymmetry, or symmetry on another axis, in the horizontal sections.

In Botta, elements enter systematically into similar relations. In Le Corbusier their distribution is multiplied into different relations. In Botta, there is a geometrical unity creating a single interpretation. In Le Corbusier, there is geometrical complexity generating a plurality of meaning. In drawing, Botta's buildings read as simple geometrical volumes that are symmetrically excavated along the back to front axis. The different systems of relations governing Le Corbusier's buildings cannot be grouped into a single principle.

An observer moving around Botta's houses sees the block as a physical entity with clearly defined vertices. He also sees a symmetrical front and congruent sides. Trying few positions and a privileged frontal view point, one can understand the geometry of the external articulation of the building as a whole. Le Corbusier's villas look different from different points of view. From the front, Villa Stein reads as a block (Table 2, figure 5.2). From the sides the block is evident but the openings revealing the terraces behind the planes question its volumetric clarity (Illustration 1). From the back, the house is dissolved into planes and volumes, each of which demands attention for itself, challenging the previous interpretations (Table 2, figure 6). The viewer has to move and engage into a careful observation, uncovering multiple relations that never resolve themselves a geometrical unity and oneness

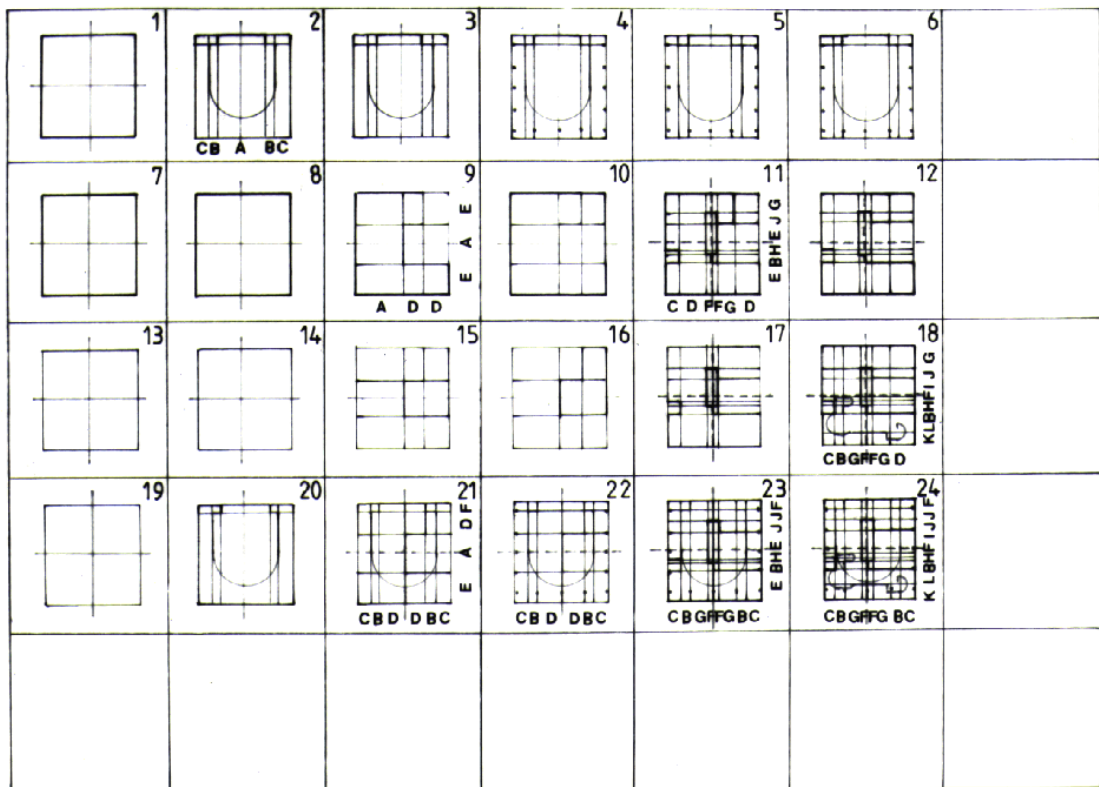


Table 7. Grid properties in horizontal sections cutting through the volume (Le Corbusier, Villa Savoie)

### 3.2 *Plan Analysis*

The plans of the houses are examined next as geometrical arrangements that are made synchronous on a flat surface. The analysis progresses again in stages from the outside to the inside, from the large to the small scale and from the last stage of the previous analysis to a detailed description of the plan.

It brings about the same observations like the ones presented previously. Botta creates a distinction between the block and the rest of the shapes throughout the stages. This is based on a definition of the corners of the block by opaque material, and on a decomposition of the corners of the smaller shapes into glazed and opaque surfaces (Table 8, figures 5, 9, 12, 15, 17, 23, 26, 29, 32 and 35). This enables the outer surfaces to extend and complete the largest shape. Le Corbusier uses both opaque and glazed material to define the corners of the block and the rest of the shapes (Table 8, figures 38, 41, 44, 47, 53, 56, 59, 65, 68 and 71). Therefore, in Botta, the physical definition of the largest component and its hierarchical distinction is constantly preserved. In Le Corbusier, the elements of the large and the small scale interact constructing a gridded pattern of lines.

In Botta, the shape organisation moves from overall symmetry at stage one (Table 8, figures 1, 4, 7, 10, 13, 16, 24, 27, 30 and 33), to asymmetry at stage three (Table 8, figures 3, 5, 9, 15, 21, 23, 26, 29 and 35). The organisation of the grids changes from overall symmetry to 'just about' symmetry (Table 9, figures 1-35). A shape or a grid configuration are defined as 'just about' symmetrical when more than seventy percent of the total number of shapes or lines are symmetrical on an axis. Therefore, shape and grid symmetry are not preserved. However, symmetry breaks by the small scale articulation occurring at the last stage. This can be seen by the way in which the symmetrical grid in figure 14 (Table 9), is transformed to an asymmetrical one. This is achieved by the introduction of two lines at the right side of the plan generated by the small scale elements introduced at this stage (Table 8, figures 14 and 15). It is suggested that deviations from symmetry generated by small scale elements do not undermine the overall pattern. They act as breaks in a continuous application of symmetry that seem to attract attention and reinforce the overall pattern.

In Le Corbusier, there is no symmetry or 'just about' of the shape and grid organisation in any stage (Table 8, 9, figures 36-51). Therefore, in Botta the shape, grid and physical properties of the plans are close associations to those of the external articulation. Rules are applied from the outside to the inside preserving the symmetries of the initial rectangle. In Le Corbusier, there is not a single rule that directs the geometrical organisation of the houses from the first to the last stage and from the volume to the plan.

### 3.3 *Spatial Analysis*

This section moves to the interiors of the houses. The aim is to see how they are experienced from the inside and how they reveal their geometrical ordering to a mobile observer. It will show that in Botta, there are global scale geometrical characteristics that remain invariant in the transformation of visual information produced through movement. These take the form of recurrent, overlapping and symmetrical visual fields that bring geometrical order into the observable level. In Le Corbusier,

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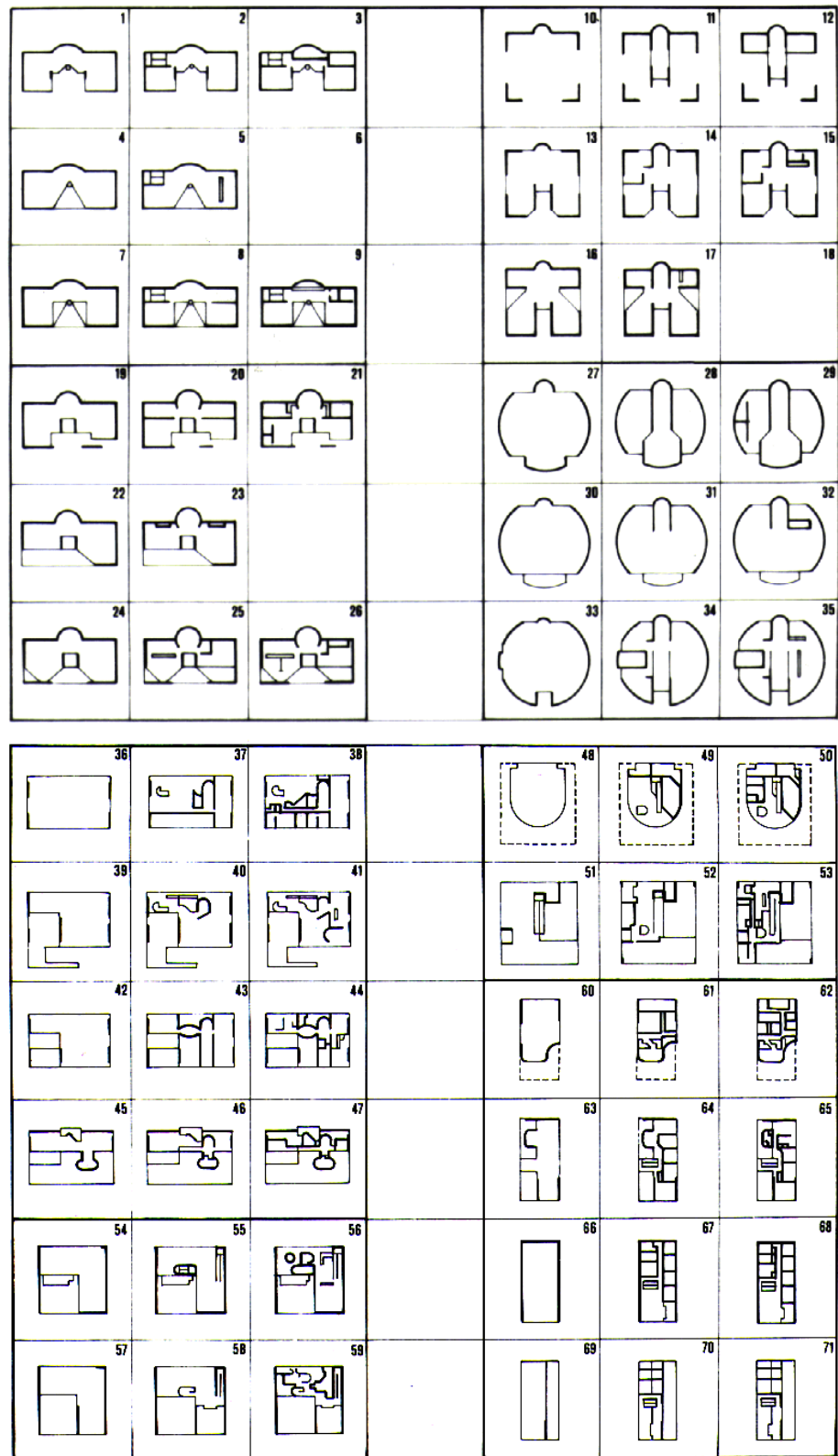
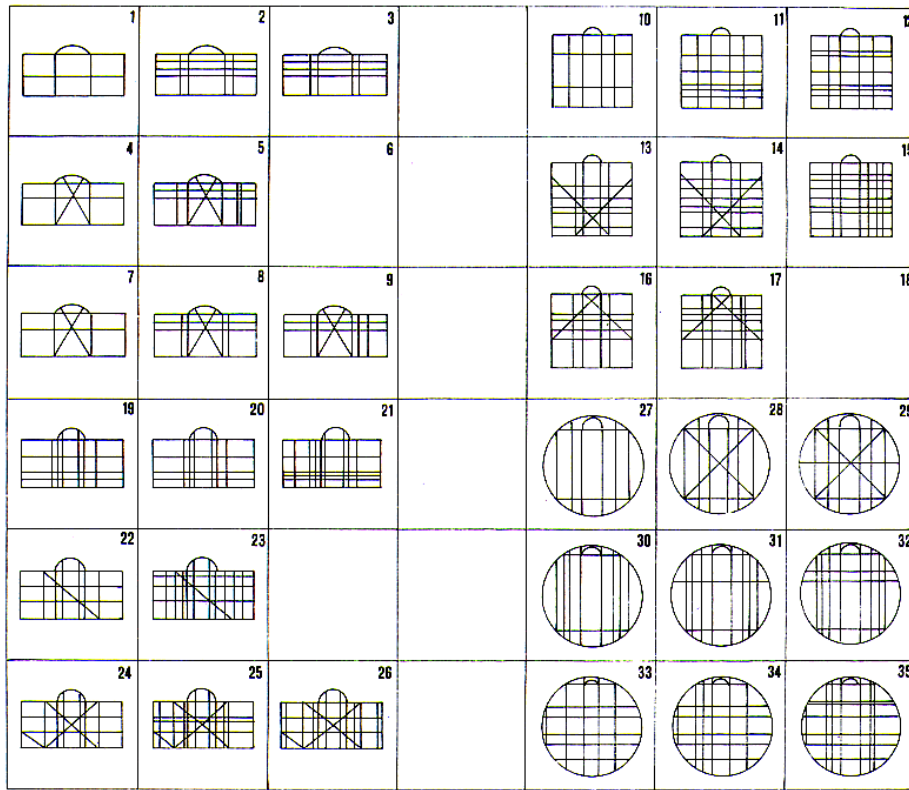


Table 8. Top-down analysis of the articulation of the plan, shape and physical properties ( figures 1 - 35, Mario Botta; figures 36 - 71, Le Corbusier).





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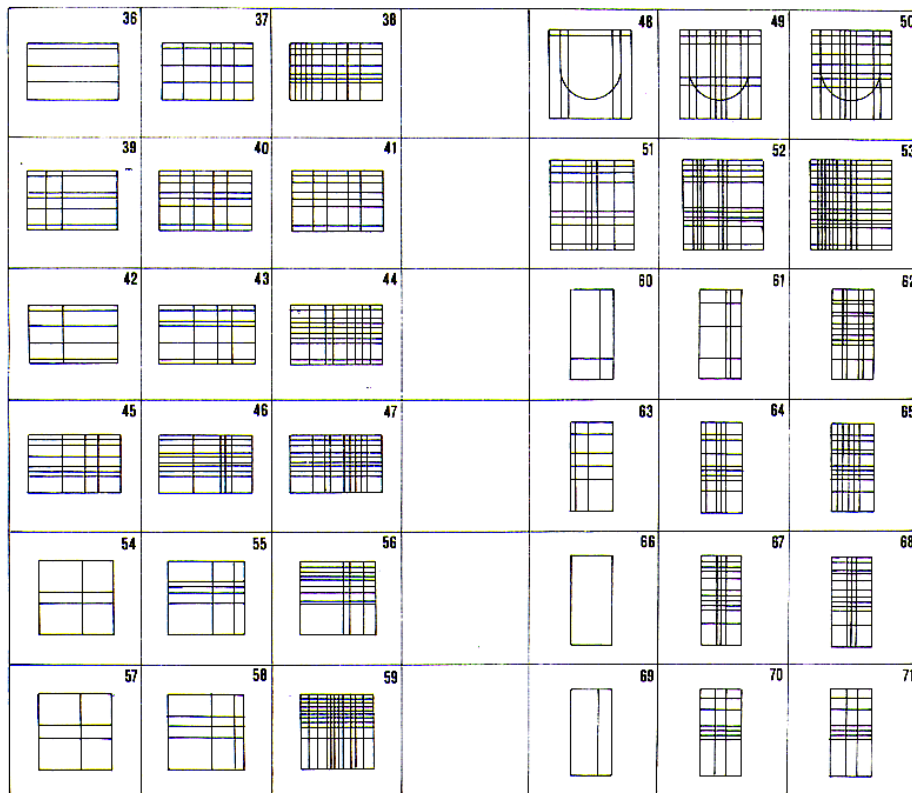


Table 9. Top-down analysis of the articulation of the plan, grid properties figures 1 - 35, Mario Botta; figures 36 - 71, Le Corbusier).

## 32.18

invariant geometrical characteristics bind local scale relations together. Besides, a lack of repetition and symmetry in visual information hide geometrical order behind a series of changing visual fields.

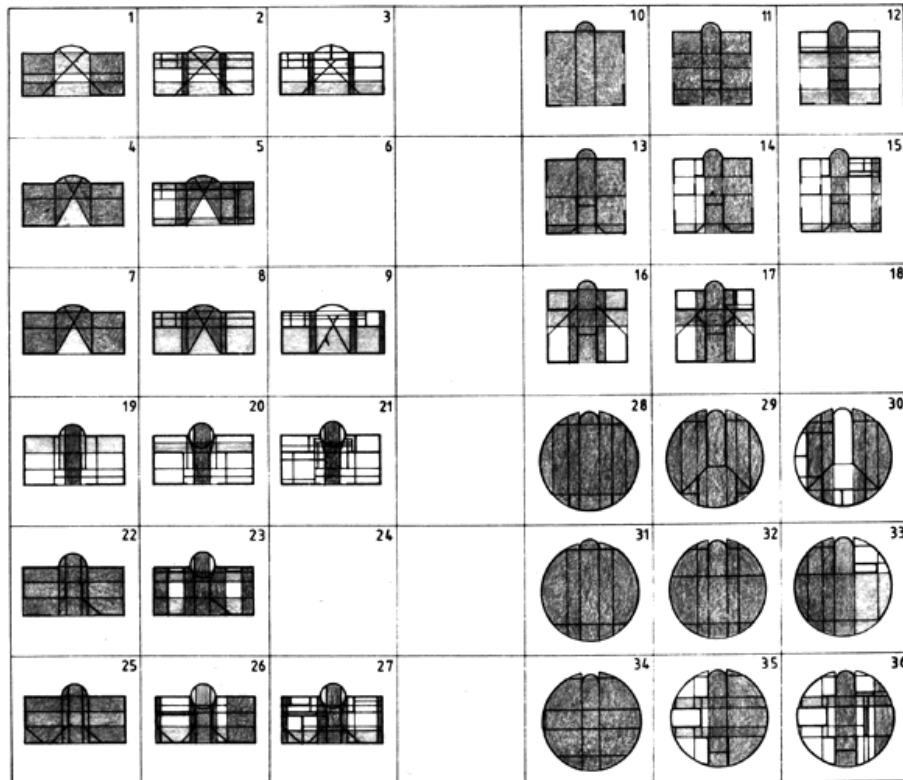
The analytical tools used here are the convex space modelling and the overlapping convex space modelling. These are applied to each of the analytical stages established by Plan Analysis. The plans are broken down to all possible convex spaces generated by the extensions of the lines defining physical elements. The convex spaces that extend throughout the width or the length of the houses (Global Scale convex spaces), are distinguished from those that do not reach the outer sides of the plan (Local Scale convex spaces), and represented on a separate table (Table 10). This is to examine the possibilities of a layout to offer large scale visual information reaching the outer sides of the volumes.

The intersections of convex spaces define overlap units (Table 11). The number of convex spaces that are synchronised by these units is expressed through a range of grey shades. White indicates a unit from which only a single convex space is seen. Analysis looks at the patterns of distribution of overlap units and the area they cover on a plan. A full coverage of a plan by units that share their defining sides constructs a continuous and successive synchronisation of convex spaces and their defining surfaces. On the other hand, a scattered distribution of units indicates that visual information is often restricted to the scale of a single convex space.

Analysis distinguishes between three kinds of overlap units. These are captured as a distinction of three shades of grey and are represented on a separate table (Table 12). The darkest shade shows an overlap unit produced by the intersection between two or more Global Scale convex spaces (Global Scale overlap unit). Medium shade stands for an overlap unit belonging to a Global Scale convex space and to one or more Local Scale convex spaces (Global-Local scale overlap unit). Finally, the lightest shade represents a third category of units (Global Scale unit). These do not result from an intersection of convex spaces. However, they belong to a Global Scale convex space, and as such, they offer information that reaches the periphery of the plan.

Visual fields are also used to represent visual experience in the layouts (Tables 13, 14). These are visual areas seen from every spatial point of a selected overlap unit (indicated by a dark shade). Radials connecting the edges of the unit with the edges of visible surfaces are excluded from this description. This is to exclude any line that does not bear a direct connection with the geometry of the layout.

The analysis of Botta's layouts shows that certain Global Scale convex spaces are preserved throughout the stages (Table 10). These are: the convex space extending at width at the centre (figures 10-12, 13-15, 16, 17, 19-21, 22, 23, 25-27, 31-33 and 34-36), those extending on either side of this space (figures 2, 3, 8, 9, 16, 17, 22, 23, 26, 27 and 28-30), those situated next to the right or the left surface (figures 4, 5, 7, 9, 13-15, 28-30, 31-33 and 34-36) and the one extending at length at the front of the plan (figures 1-9, 11-15, 22-27 and 31-33). Therefore, a large degree of global scale information and the positions from which this is offered is constantly retained. According



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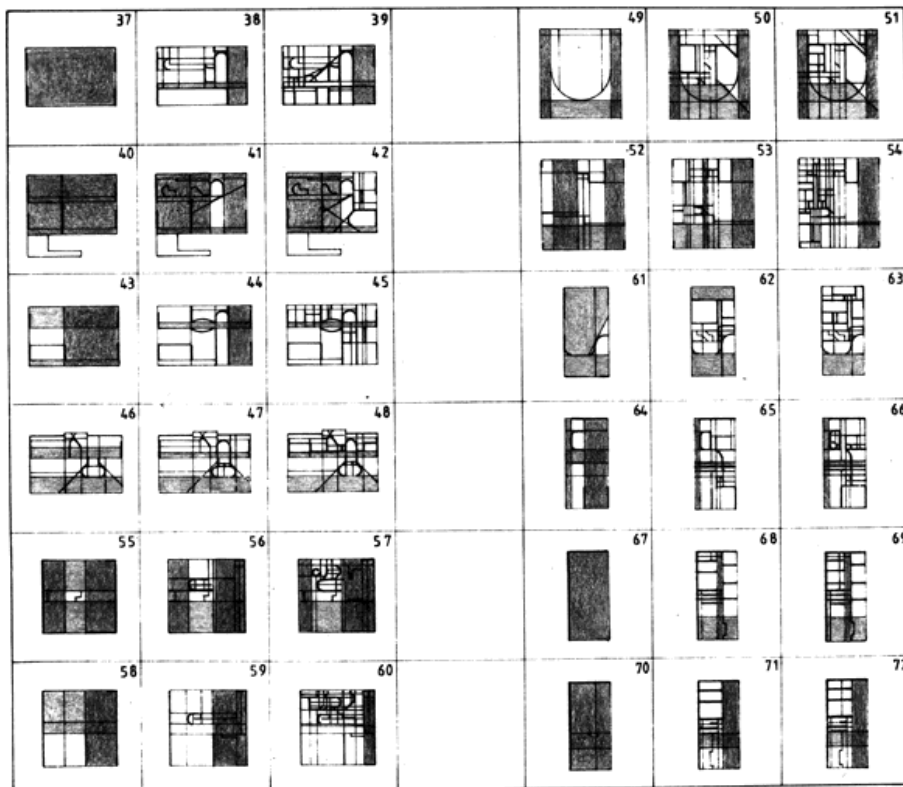


Table 10. Global scale convex spaces. These are spaces extending throughout the width or the length of the plan (figures 1 - 35, Mario Botta; figures 36 - 72, Le Corbusier).

## 32.20

to the definition of convexity, this information remains invariant in every step taken inside these spaces.

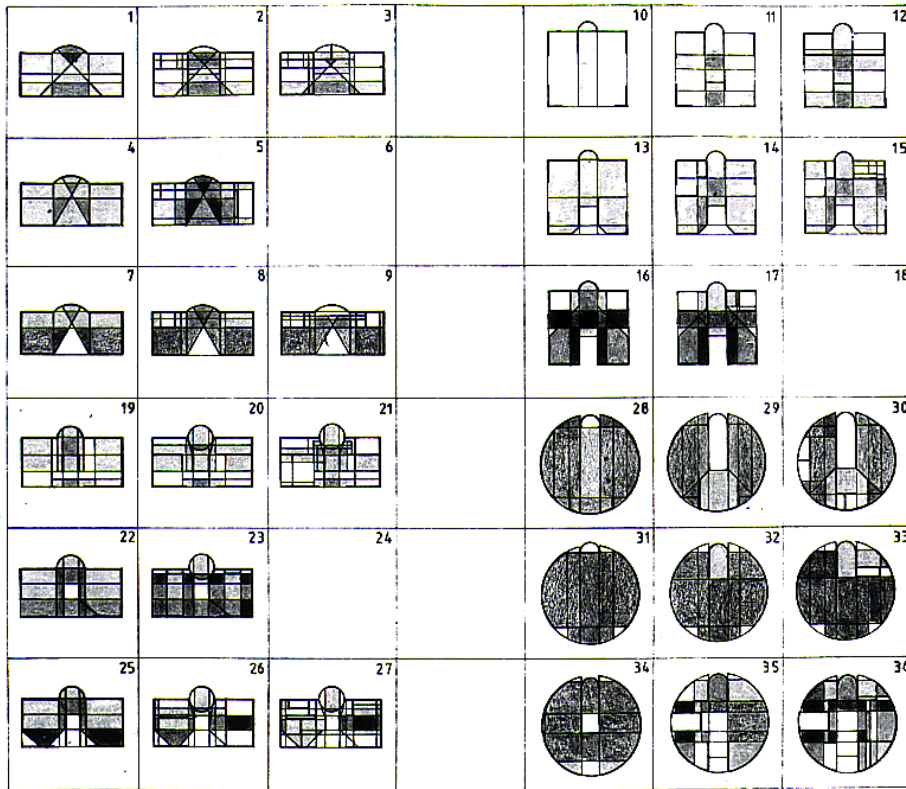
A number of overlap units also remain the same in terms of shape, size, position and overlap value in all stages (Table 11, figures 1-36). Overlap units cover a large area of the plans sharing their defining sides. A large number of these units are Global Scale overlap units and Global-Local scale overlap units (Table 12, figures 1-36). Therefore, moving in these layouts, one experiences constantly a successive synchronisation of global and local scale relations. Besides, the first floor of H1, H3 and H4 and the ground floor of H4 are exposed almost as wholes from the central overlap unit throughout the stages (Table 13, figures 4, 5, 14, 15, 22, 23, 32 and 33).

Thus, transformation retains certain spatial elements, and their distribution on the plans. It seems that it is constantly directed towards specific patterns of spatial exposure. These are: simultaneous exposure of a large part of the layout (from the central unit), constant successive synchronisation of global and local scale spatial relations (from the overlap units) and global and local scale information that stays invariant over a number of steps (from the Global Scale convex spaces and the overlap units).

There is also an association between certain patterns of exposure, certain spatial locations and certain floors. This is because Global Scale convex spaces are situated always at the centre, the sides and the front, exposing global scale information from the same positions. It is also because, the first floor is revealed as a whole from the central spatial point, as opposed to the ground and the second floor in which a large area of the plan is seen from the front through the glazed surfaces of the void. This generates a repetitive pattern of information transmission along the horizontal and the vertical direction that contributes to the intelligibility of the layout, and the house as a whole.

The effects of the above characteristics to the constitution of visual fields by surfaces are as follows: there is visual access to the entire length of the front, the right or the left, or both the right and left outer surfaces of the layouts from the Global Scale convex spaces; there is simultaneous synchronisation of the outer boundary and the internal partitions from the central unit; there is continuous and gradual synchronisation of these elements from the global and local scale overlap units; the outer and the inner surfaces enter visual fields, binding close and distant spatial elements together. *Thus, there are global and local scale physical characteristics that remain invariant in the transformation of visual fields produced by movement.*

A large number of Global Scale convex spaces, Global Scale units and overlap units, are symmetrical on the back to front axis in all stages. Thus, there are symmetrical or almost symmetrical visual fields constructed from the units situated on the axis (Table 13, figures 3, 5, 9, 12, 15, 17, 21, 23, 27, 30, 33 and 36). Besides, there are symmetrical or almost symmetrical visual fields released from symmetrical positions (Table 14, figures 1-2 and 3-4). Therefore, the repetitive association between visual fields and certain locations is enhanced by symmetrical visual fields. *Repetition and symmetry shows that there are invariant geometrical characteristics built into the system of visual information.*



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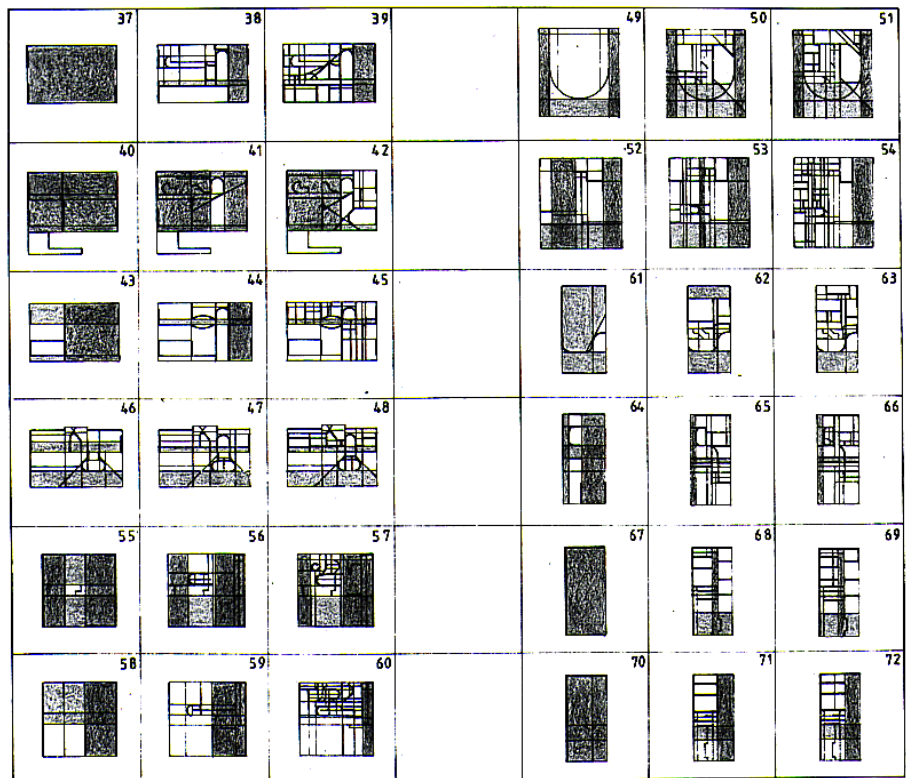


Table 11. Overlap units. Spatial nits defined bby the intersection of convex spaces ( figures 1 - 35, Mario Botta; figures 36 - 72, Le Corbusier).

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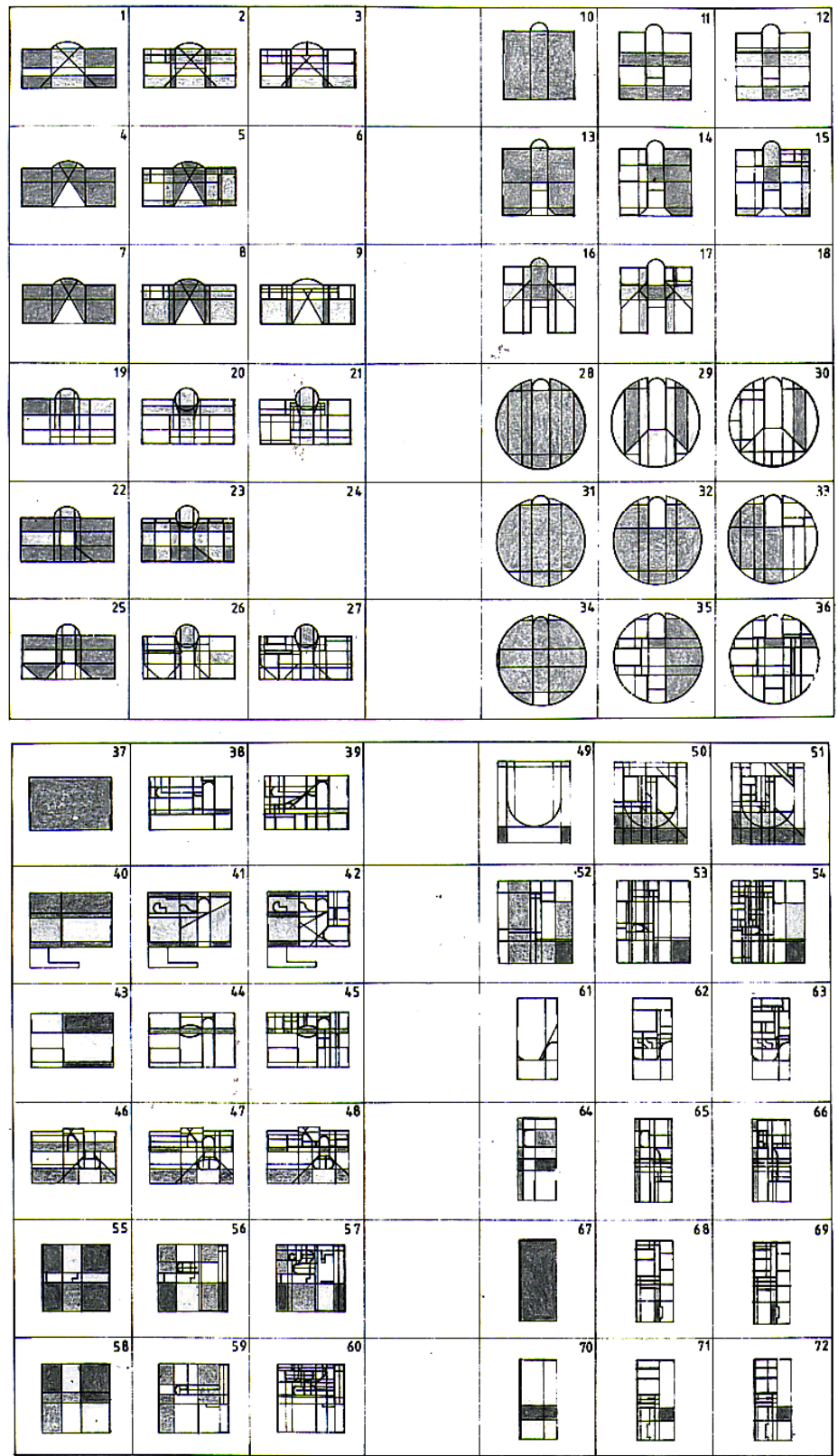


Table 12. Global scale overlap units. They are defined by the intersection amongst two or more Global Scale convex spaces or by the intersections amongst a Global Scale convex space and one more Local Scale convex spaces. The latter are convex spaces that do not reach the periphery of the plan (figures 1 - 35, Mario Botta; figures 36 - 72, Le Corbusier).

Moving to Le Corbusier, Table 10 shows that certain Global Scale convex spaces extending along one side (figures 38, 39, 40-42, 49-54, 55-60, 64-66 and 70-72), and along the front of the plans (figures 40-42, 46-48, 49-51, 55-57, 61-63, 68 and 69), are preserved throughout the stages. However, from stage one to stage three, the number of these spaces and the number of the Global Scale overlap units resulting from their intersections is decreased (Table 12, figures 37-72). This indicates a decrease in the number of views that reach throughout the plans, as well as in the number of surfaces that are seen in full length.

32.23

From stage one to three, the number of overlap units becomes larger, while their size becomes smaller (Table 11, figures 37-72). Most of these units are marked by light shades of grey indicating a small number of convex spaces to which they belong. They also form small clusters that are isolated from each other by intervening non overlap units and boundary walls. It seems that the majority of the visual fields are restricted to the local scale of three, two or a single convex space. These fields offer access to different convex spaces or to different bounded areas (Table 14, figures 49-95). Therefore, there is a limited synchronisation of convex spaces and their defining surfaces and a large number of restricted visual fields that are different from each other.

However, in some layouts, there is a large cluster of overlap units that share their defining sides in all stages. This is situated either at the open spaces or it extends from the exterior to the interior through the glazed surfaces of the terraces (Table 11, figures 42, 48, 51, 54, 57 and 66). Some of these units are Global Scale units (Table 12, figures 42, 48, 51, 54, 57 and 66). Thus, a certain degree of synchronisation of global and local scale relations, and of constant visual contact with certain areas and

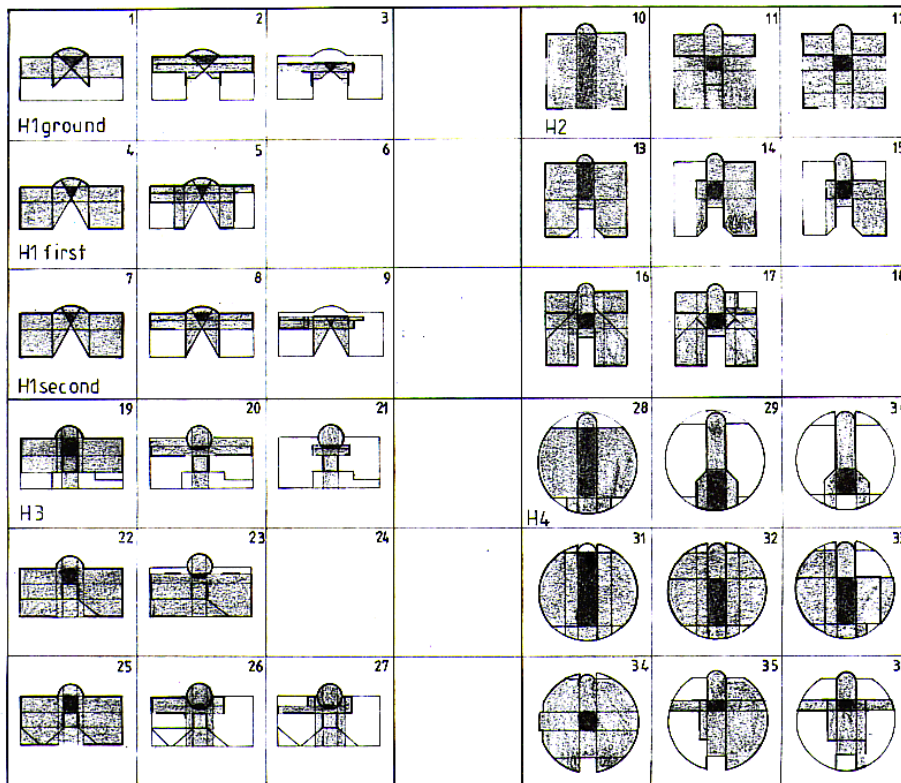


Table 13. Visual fields from the central overlap unit ( Mario Botta)

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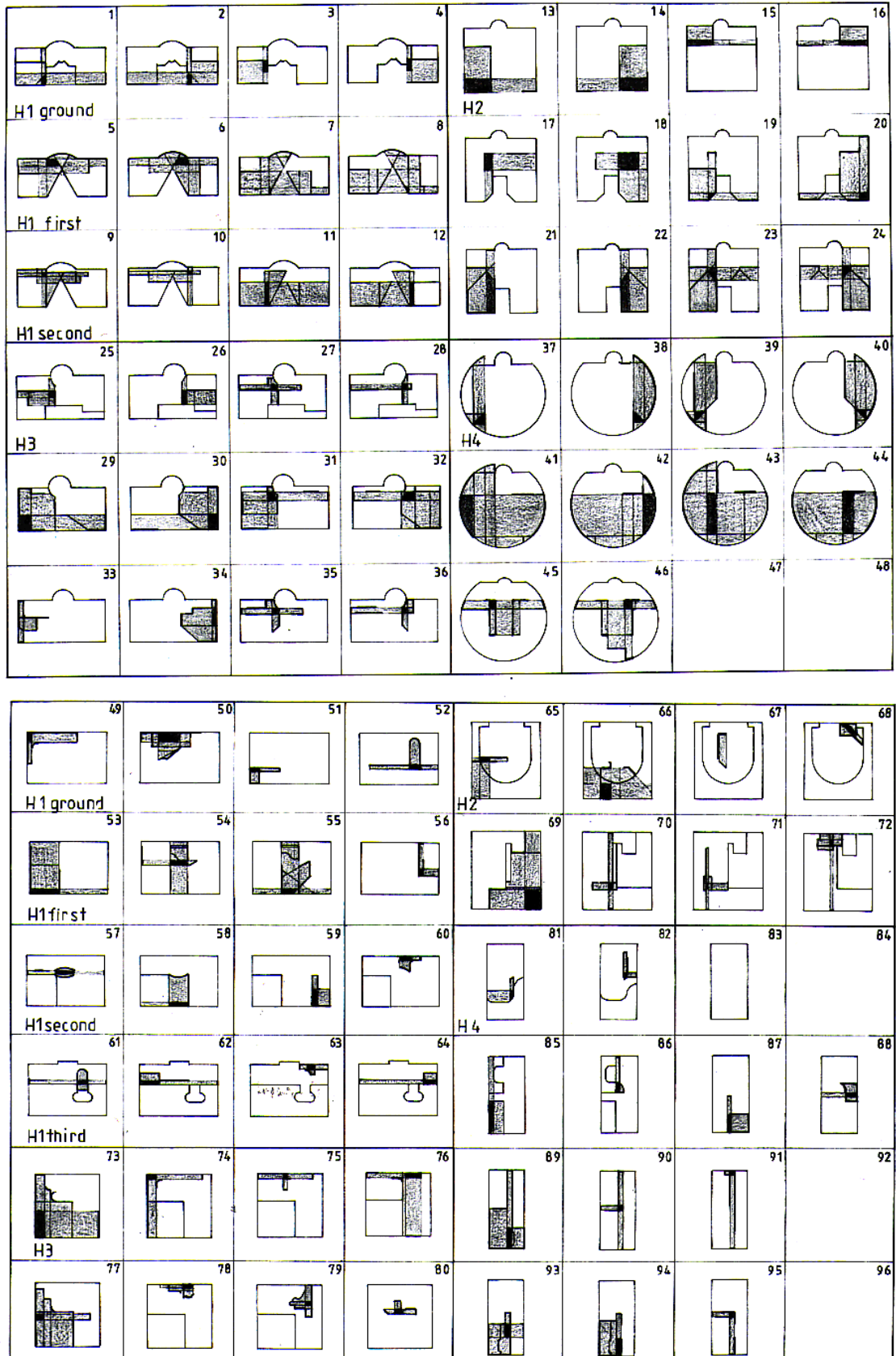


Table 14. Figures 1 - 46: visual fields from overlap units that are symmetrical on the back to front axis (Mario Botta); figures 49 - 95: visual fields from overlap units that are closely situated or are in distance from each other (Le Corbusier).



surfaces is always preserved. It seems that Le Corbusier creates a distinction between expansive and restricted views. From the former, vision enables a stationary appreciation (Table 14, figures 55, 66, 69, 73, 76, 77 and 89). From the latter, it embraces very little at a time (Table 14, figures 49, 51, 52, 56, 59, 60, 63, 71, 72, 78-80, 81, 82 and 86-88). Contrasting moments of expansion and contraction, of openness and closeness, of visual exposure and intricate Local Scale articulation construct a rhythm of permanence and rapid change in visual reception. This rhythm makes the experience of the layout a journey through different spatial episodes.

32.25

There is no association between certain kinds of views and spatial locations. Although expansive views are maintained, the positions from which these are offered varies from floor to floor and from house to house. At the ground floor of LH2, for example, large scale views that connect the inside with the outside are constructed from the front of the layout (Table 14, figure 66). At the first floor, they are transmitted from the right side of the plan (Table 14, figure 69). This makes the experience of each individual level, and of each house, different from the others.

There is no symmetrical distribution of spatial elements either. This shows that there is no regularity embedded into the structure of visual information that can co-ordinate distant and close visual fields into a geometrical pattern.

#### 4 Space, geometry and intelligibility

The discussion moves next to the ways in which the geometrical properties observed during the sequential experience of these layouts, reveal the geometry of the three dimensional and the two dimensional organisation of the buildings as wholes.

In Botta, there is a simultaneous and a successive synchronisation of Global and Local Scale physical relations. There is also a repetitive and symmetrical pattern of information transmission. The outer boundary enters a large number of visual fields which are symmetrically organised with respect to the back to front axis (Table 15, figures 1-21). *Therefore, there are Global Scale invariant geometrical characteristics in the form of the outer surfaces and this axis in the transformation of visual information.* It was suggested that in the transformation of the volume and the plan, the properties of the block remain also invariant, gathering all elements under their coordinating role. *Therefore, the same rules are hierarchically applied from the higher to the lower level of abstraction, and from the geometrical structure of the buildings as a synchronous system, to their structure as a sequence of visual fields.*

In Le Corbusier, there are areas in which the outer boundary remains invariant in a number of spatial steps (Table 15, figures 31 and 32). However, from the largest part of the layouts, visual fields reveal surface interconnections at the local scale of one to three convex spaces (Table 15, figures 23, 27, 28, 29, 30, 34 and 36-40). Besides, there is no symmetry built into the structure of visual information. In other words, there are only Local Scale invariant geometrical characteristics picked up through motion. It was argued that the transformation of the volume and the plan preserves and negates the hierarchical distinction of the block and its axis. Therefore, both the synchronous organisation of geometry, and the sequential experience of space, question the role of the largest shape to organise relations at a Global Level.

## 32.26

*These observations seem to lead to the suggestion that in Botta spatial experience reveals the geometrical order of the buildings. In Le Corbusier, geometrical order is not immediately available, hidden behind the asymmetrical and constantly changing visual sequences.*

The question that arises next is: *how these houses become intelligible and which is the role that an exposed or a hidden geometry plays in this intelligibility?* To answer these questions, the different ways in which the two categories of buildings are experienced, are discussed further.

#### 4.1 *Static-dynamic spatial experience*

It was suggested that Botta creates an immediate or a successive exposure that presents maximum information to an almost static observer. Le Corbusier creates a delayed exposure that sequentially reveals small scale information to a peripatetic observer. The former constructs a static experience. The latter creates a dynamic one.

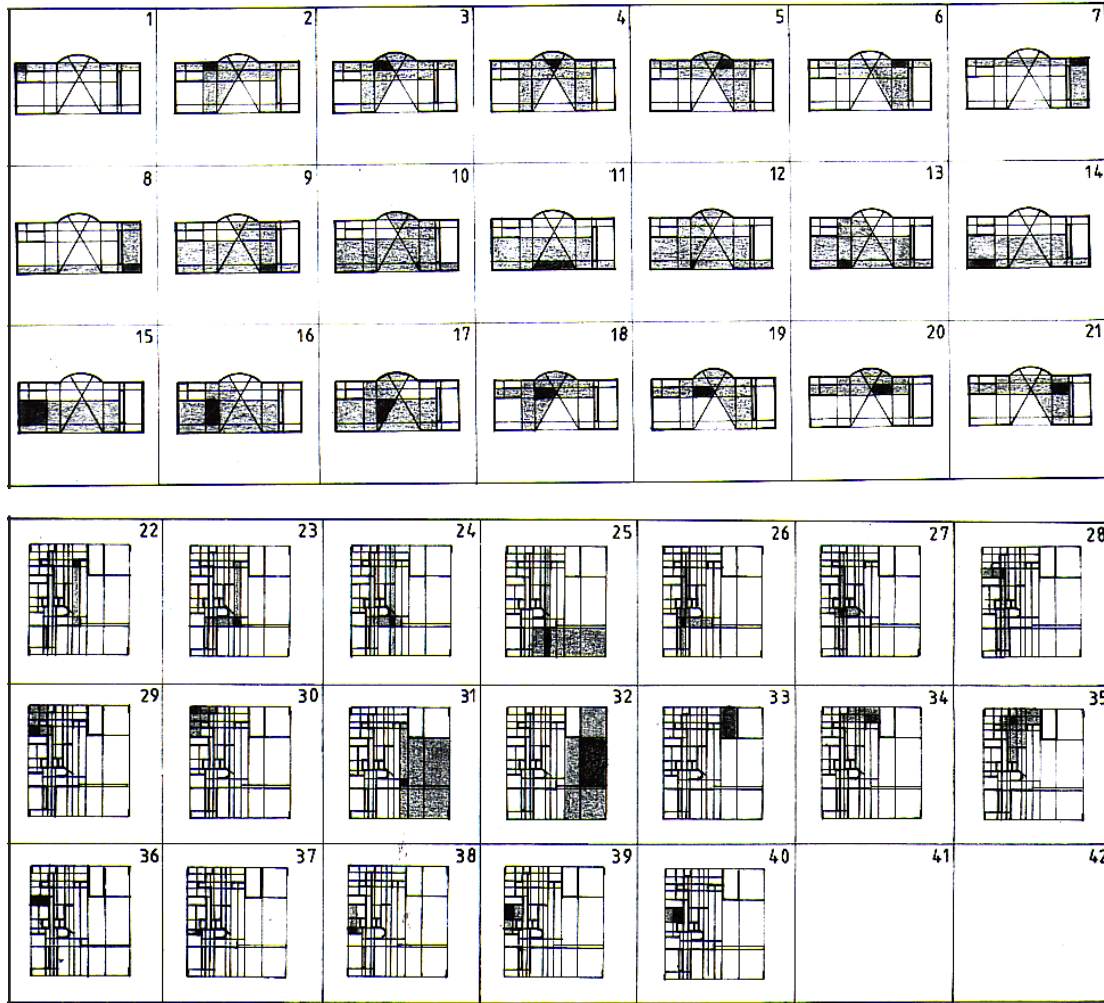
Botta's layouts are grasped from a single or from limited positions. To grasp Le Corbusier's houses one has to put together the changing and intricate spatial events encountered step by step. Intelligibility in Le Corbusier is based on a cognitive synthesis demanded from the viewer. In Botta, it is based on a visual synthesis offered to the viewer. However, the ground and the second floor of Botta's houses are not immediately available. Therefore, a cognitive synthesis is required in his case also. The question to ask then is: *how is this synthesis achieved in both categories of houses and according to which operations?* To answer this question, the discussion moves to some other characteristics of spatial experience in these layouts.

#### 4.2 *Probabilistic-deterministic spatial experience*

In Botta, the repetitive and symmetrical pattern of visual exposure shows that a certain order is built into the transformation of information. In Le Corbusier, there is less order characterising the occurrence of visual fields. The probabilities of what is encountered in each step and in which way are open. Thus, spatial experience in Botta accommodates little probability and a great deal of repetition and certainty. In Le Corbusier it accommodates no repetition and a great deal of probability and uncertainty. The former is a *deterministic* experience. The latter is a *probabilistic* one.

#### 4.3 *Continuous-discontinuous spatial experience*

In Botta, each visual field overlaps with the previous and the next one (Table 15, figures 1-21). Each position offers a fragment of visual information that is offered before. In Le Corbusier, there are certain areas in which visual fields retain constant access to certain portions of space (Table 15, figures 22-25). There are also areas which either lose contact with a global reference or have nothing in common (Table 15, figures 22, 27, 29, 31, 33, 34, 36 and 37). In Botta, visual information changes smoothly. In Le Corbusier there are smooth and sharp changes in visual fields. Spatial experience in Botta is based on *continuous change*. In Le Corbusier, it is founded on alternating patterns of continuous and *discontinuous change*.



32.27

Moving inside Botta's houses, a viewer might realise which positions to occupy to see most, and which to omit to avoid repetition. A repetitive and continuous pattern of information transmission gradually generates a certain knowledge about the layout and the house as a whole. There is a 'forward walking' (Gombrich, 1984), that enables assumptions about the global system from the invariant characteristics that are observed locally. In Le Corbusier, it is not possible to know what the layout looks like prior to exploration. A probabilistic and discontinuous experience creates a variety of possible interpretations. It is only when all visual fields are tried and spatial positions are exhausted that comparative knowledge is achieved.

Table 15. Figures 1 - 21: sequences of visual fields in Botta's house at Viganello; figures 22 - 40: sequences of visual fields in Le Corbusier's Villa Savoie.

Thus, Botta enables the viewer to anticipate in advance what visual fields offer, and from which locations. These anticipations are constantly validated through repetition and continuity, closing down the probable hypotheses and leading to a final interpretation. In Le Corbusier, hypotheses are more open because there are many alternatives in the ways in which information is revealed. Fulfilled predictions generate forward readings. Unfulfilled ones make a viewer readjust his expectations. *Intelligibility in Botta seems to develop according to a closed and predictable sequence of mental operations based on confirmed hypotheses. In Le Corbusier, it develops according to an open and unpredictable sequence of operations based on confirmed and disconfirmed hypotheses.*

It could be suggested that Botta enables the visitor to take 'inferential walks' (Eco, 1994), outside space, and use his mental operations to understand the ways in which spaces are held together from a distance. Le Corbusier seems also to enable inferential walks. However, the mental processes are challenged and the visitor is required to slow down and take actual walks in the sequential medium of space.

It seems that in Botta the viewer's attention is directed from the sequential unravelling of space to an ordered pattern of repetition, symmetry and binding surfaces that organises it at a geometrical level. In other words, Botta enables the viewer to direct and organise his spatial experience by geometrical properties. Le Corbusier, prolonging sequential experience, attracts the viewer's attention to this experience. At the same time delaying access to a final and single interpretation about how the spatial sequences relate to each other at the overall geometrical level, he arouses his keener interest in this interpretation. It seems that space systematically breaks and delays access to the properties of geometry, arousing keener interest in it.

Geometry in Botta provides a viewer with a reference to understand the narrative of movement. In Le Corbusier, geometry is hidden, providing the viewer with a network of open interpretations to this narrative. In Botta, intelligibility is based on a clear and single pathway to knowledge. In Le Corbusier, knowledge is unlimited, assuming the form of an open and probabilistic network of pathways and a continuous interrogation.

## **5 Conclusion: geometry, space and composition**

At this point a brief reminder of the main arguments of this paper is offered. It is suggested that in Botta's Global Scale invariant geometrical patterns observed through movement raise geometrical order into the observable level, and structure intelligibility under a single interpretation. In Le Corbusier, Local Scale invariant geometrical characteristics picked up in motion, hide geometry leading to a network of multiple interpretations. In this respect, the examination of the two architects seems to reaffirm the initial hypothesis. It is through a study of invariant geometrical properties in the transformation of visual fields that the relation of geometry to space can be explored.

This analysis leads to another important observation: the two different kinds of intelligibility depend on two different kinds of transformation. In the first one, there is a systematic preservation of properties, from an abstract to a specific state, and from geometry to space. In the second one, there is a preservation and suspension of these properties. In this way, the top-down analysis progresses not only from geometry seen in space to geometry as a synchronous organisation represented in drawings but also in a reverse order - from the ways geometrical relations constructed in drawings, create specific patterns of exposure in space.

Designs are generated by abstract geometrical definitions which are recursively transformed and refined into detailed drawings. It seems that the top-down approach studying geometrical systems as transformations from abstract to detailed stages approaches the logic and the operational strategies of a design process.

Considering the strategies and the design logic of these architects, it was shown that Botta employs a clear syntax based on a single way of transformation. Le Corbusier employs an implicit syntax and a variety of combinations. Botta reduces the combinatorial possibilities in the articulation of geometry and space using a single rule. Le Corbusier opens the field of possibilities through a suspension and inclusion of rules. In Botta, the rules are supplied from outside the transformation. This is because they are about a simple geometrical concept that is known prior to any design combinations. In Le Corbusier, the houses are shaped through external rules of a single solid, and through internal ones that decompose this solid and emerge during the process. Thus, whereas Botta structures the transformation to satisfy pre-existing knowledge of a building form, Le Corbusier directs it to release combinatorial possibility and a potential energy in form. The former is a deterministic design logic. The latter is a probabilistic one.

32.29

Extending these observations further, it seems that the design logic of the two architects has a connection with the ways in which their buildings are grasped, i.e. in a deterministic and ordered way leading to a single interpretation (Botta), and in a probabilistic and open way generating multiple meanings (Le Corbusier). It could be argued that the ways in which these buildings communicate their structure to an observer is analogous to the ways in which they communicate their structure to the designing architect. Thus, the architect and the viewer are complementary entities, constructing each other in the process of making and in the process of viewing.

It should be noted that these observations are made in the limited context of the selected buildings. Their validity is subject to further investigation which must open to include a wider sample. However, within these limitations, a final proposition is made: an approach to geometrical and spatial description must be a top-down approach releasing the combinatorial logic of a genesis - the logic of composition as a dynamic activity in process.

#### References

- Arnheim R., 1974, 'Art and Visual Perception', California University Press, p. 248.  
 Benedict L. H., 1979, 'To Take Hold of Space: Space and Isovist Fields', Environment and Planning B, Volume 6, pp. 47-65  
 Eco U., 1979, 'The Role of the Reader', Indian University Press, p. 49  
 Eco U., 1994, 'Six Walks in the Fictional Woods', Harvard University Press, p. 50  
 Gibson J. J., Cornsweet, J., 1952, 'The Perceived Slant of Visual Surfaces: Optical and Geographical', Journal of Experimental Psychology 44: II-15  
 Hagen M. 1986, 'Varieties of Realism', Cambridge University Press, pp. 33-63  
 Hillier B., Hanson J. 1984, 'The Socia Logic of Space', Cambridge University Press, p. 98