

**URBAN CONFIGURATIONAL COMPLEXITY***Definition and Measurement*

## 05

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**0 Abstract**

Among the several notions and expressions used in urban morphological description and analysis, *complexity* is a very common one. Nevertheless it has been used in a quite loose and fuzzy way, never getting enough definition and precision. In this sense, the expression, instead of clarifying descriptions, helps to actually blur them. This paper reports an attempt to give the expression *complexity* a more precise meaning, and by doing so, offering the possibility to describe and measure it in a more rigorous way, within the boundaries of urban configurational studies.

It is assumed an urban system is formed by public spaces and built form units, both described by a number of discrete entities. Both basic components are related to each other through adjacencies, so that the system can be expressed by a graph. Considering that each BFU is reachable from every other through a path formed by public spaces, it can be said that within the system there are a number of virtual spatial interactions between pairs of BFUs, distributed among a number of minimal paths linking them.

In this context, it is proposed the concept of complexity as a function of the number of possible pairs of BFUs, on the one hand, and of the number of minimal paths between each pair, on the other. To that extent, complexity is related to quantity and distribution of stocks, and to connectivity of public space network. Maximal complexity is attained by a system configuration in which the stocks distribution gives the maximum number of BFU pairs and the public space network gives the maximum number of minimal paths throughout. From the definition is derived a measure of complexity which offers in a single figure an idea about how a given system is positioned in a known scale.

**1 Introduction**

The intuitive notion of complexity has been largely used in urban studies in two different ways: firstly, within the context of social sciences, as reference to a perceived superimposition of domains, systems, approaches, causes and effects of the urban matter. In this way, complexity seems to be related to the difficulties of having a clear cut urban problem definition: environment, built space, society, politics, economics, production, reproduction and accumulation are closely related to each other, turning each one's output effect from the action of the others, as well as cause to all others. In this context, complexity is related to conceptualisation and description of large, diverse and complicated systems of many variables. Secondly, the word complexity is used as reference to supposedly essential qualities of urban form/life. It is said that city form/life differs from, say, suburbs precisely for its higher degree of complexity. In this sense, complexity here is a value, belonging to the domain of evaluation. Urban designers and urban morphology researchers refer to complexity as an urban property, related to at least one of the concepts of variety,

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scale (size), growth, intensity, continuity, density. Hence, complexity could be understood as a particular state of urban form/life, in which one or more of those aspects are present in a great extent, or beyond a given threshold.

Although largely quoted, the concept of complexity still is poorly implemented in terms of precise description and measurement. Authors can hardly agree upon a common conceptual core for the expression; the very use of terms such as variety, size, density, intensity of usage, etc., as proxies and components of complexity gives an idea to what extent its definition can vary and encompass a range of variables, mixing the objectual, spatial, functional and symbolic dimensions of the city.

Configurational studies can possibly offer a good background for a more precise definition, to the extent that it treats the spatial problem in a rigorous way, in connection to its social logic. It should be possible and beneficial to define *spatial complexity*, that is to say, to define the spatial components of urban complexity. Configurational studies are referred to in this text as the field of urban space study that deals with particularities of urban form. Space has been usually taken in main stream urban studies as distance. Urban economics, as well as spatial interaction reduce space to a friction restricting any social interaction. Urban configuration, alternatively, intends to take space in a little more detailed way and look at its inner structure. Space syntax, as specified below, is a method for configurational analysis within which complexity can be initially examined. Further development is obtained through introduction of centrality studies.

## 2 Space, syntax and complexity

Urban space allows a variety of descriptions, the most common ones being photographs, maps or drawings. Configuration is a spatial description too; nevertheless it differs from the former ones in the sense that it is a systems description of urban space. A urban space configuration is a representation of the urban spatial reality given by a few categories of components and rules which tie each component to all others in such a way that a change in any one of these basic elements reflects on the entire system. In this sense, an axial map, for instance, together with a connectivity matrix describes a configuration of an urban layout, based on components - lines - and a rule - adjacency. A survey map would not be a configurational description of the same urban area, yet several components of the map could be taken off without impact on the remaining ones.

From the definition above, configuration is a state description of urban systems and could either be taken statically as such, that is, a sort of layout "x ray" - the analogy seems plausible, to the extent that configurational descriptions are rather abstract - or dynamically, in this case representing a particular state of a process. A configurational description also gives the opportunity to measure particular quantitative properties of the system represented; connectivity indexes, asymmetries, spatial opportunities, are all measurable properties of a spatial configuration. Complexity can be thought of in the same terms, as being a particular property of spatial configuration, however it is important, first, to have a look at how spatial configurations have been described in different theoretic systems. Different systems' representation would prompt different definitions and domains for the measure of complexity.

Space syntax is said to be a method for spatial description and analysis; once applied in an urban context it would account for some basic characteristics and properties of urban layouts. Description is obtained through the reduction of urban space to just one spatial category - the axial line (Hillier & Hanson, 1985; Hillier, 1994; Hillier, 1996) - which is a very economic representation of public urban space, since it retains, along with the linear dimension of the street web, the fundamental property of connectivity. So, the system description underlying most syntactic studies is constituted by one spatial category, which is precisely the axial line, and one relational rule, the adjacency. Although several configurational measures could be taken from it, Space Syntax relies heavily on the measure of relative asymmetry - RA. RA adapts the concept of accessibility, retaining the notion of shortest path, replacing distance by depth and the overall sum by the average. Accessibility is a measure of relative position and can be assessed through a variety of procedures; the analogue of RA is the simplest one; it considers that one location, within a system of locations, is best accessible if the sum of the distance from it to all other locations is lowest. In any case it is assumed that the distance between any two locations is always taken along the shortest possible path linking them. Usually accessibility models take distance in terms of length, time or cost of transportation between two locations; the RA model introduces the notion of depth, which is a sort of topologic distance. Considering a graph, the depth of a point, in relation to other is given by the number of nodes included in the shortest path between these two points. If they are adjacent, the depth of one to the other is one. Accessibility indexes are usually represented by the sum of the distances from one location to all others; the RA model takes the average value.

Relative Asymmetry is computed by taking the average depth of a considered space to all others, relativised in such a way that the results vary within a 0-1 scale. RA is fundamentally a measure of spatial differentiation that ranks up all systems' spaces according to their particular position in relation to all others. *Integration* is the word chosen to name this spatial property. Integration, although being a standalone configurational measure, is frequently compared to patterns of urban pedestrian flows; significant correlation between the two series of data is assumed as evidence of a certain social logic of space. This means that space configuration and social behaviour do have a common interface. Society, through space configuration, works out patterns of social interactions characterised by non controlled social encounters occurred in the realm of urban public space. In this sense, space would be a sort of machine able to prompt/prevent social interaction, according to peculiarities of the layout. Patterns of integration and segregation would arise from the very articulation of streets in the city.

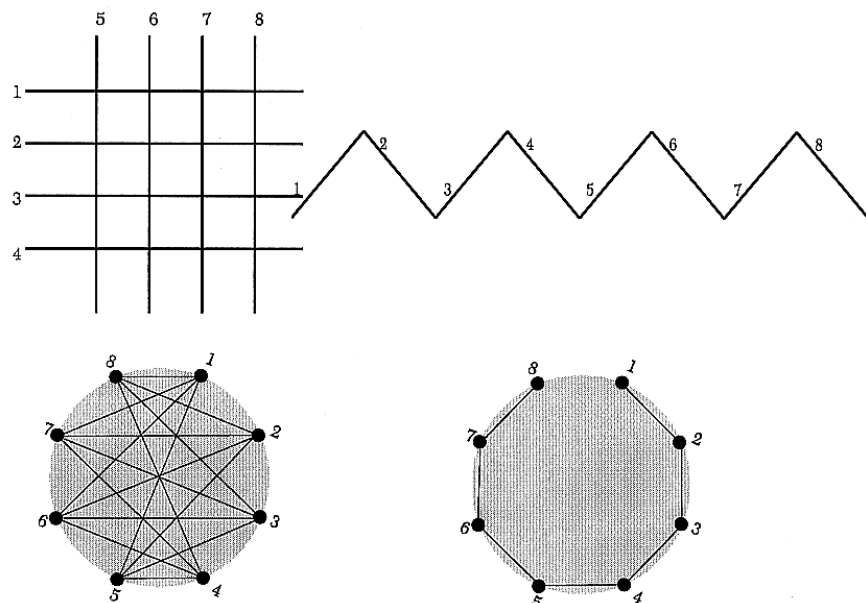
It is within this framework that complexity should be addressed, and it can be done in two alternative ways: a) complex would be the configuration which elaborates the spatial articulation (adjacencies) most, resulting in a highly hierarchised spatial system with reduced and controlled patterns of social interaction; b) complex would be the configuration that elaborates the spatial articulation least, resulting in a pattern of high social interaction and a very distributed system. The expression *spatial articulation elaboration* is used here in a restrictive sense, that is, the most elaborated spatial articulation is the one in which the occurrence and distribution of adjacencies is more restricted, resulting in deep spatial configurations. The first interpretation

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points toward the tree shaped spatial configurations; the second, to grids. Trees are highly hierarchical spatial systems, with key spaces controlling the access to many others and making the path pattern very concentrated. Grids, in the opposite position, are open systems in which each space is virtually no controlled by any other and the pattern of flows is very distributed.

Both situations can be assessed by looking at the RA index span: the “a” case is denounced by a large interval between the higher and the lower RA value; the “b” case by small difference between the two extreme figures. It means that the first situation presents some deeply segregated spaces; the second case shows all spaces in virtually the same integration position.

Although both above alternative definitions of complexity look, in principle, plausible, **the second one** seems better rooted in the theory that underlies systems and Space Syntax, as follows:



| GRID<br>SPACES 1 TO 8 |       | "TRAIN"<br>SPACES 1 and 8 |       | "TRAIN"<br>SPACES 2 and 7 |       | "TRAIN"<br>SPACES 3 and 6 |       | "TRAIN"<br>SPACES 4 and 5 |       |
|-----------------------|-------|---------------------------|-------|---------------------------|-------|---------------------------|-------|---------------------------|-------|
| PAIR                  | DEPTH | PAIR                      | DEPTH | PAIR                      | DEPTH | PAIR                      | DEPTH | PAIR                      | DEPTH |
| 1-2                   | 2     | 1-2                       | 1     | 2-1                       | 1     | 3-1                       | 2     | 4-1                       | 3     |
| 1-3                   | 2     | 1-3                       | 2     | 2-3                       | 1     | 3-2                       | 1     | 4-2                       | 2     |
| 1-4                   | 2     | 1-4                       | 3     | 2-4                       | 2     | 3-4                       | 1     | 4-3                       | 1     |
| 1-5                   | 1     | 1-5                       | 4     | 2-5                       | 3     | 3-5                       | 2     | 4-5                       | 1     |
| 1-6                   | 1     | 1-6                       | 5     | 2-6                       | 4     | 3-6                       | 3     | 4-6                       | 2     |
| 1-7                   | 1     | 1-7                       | 6     | 2-7                       | 5     | 3-7                       | 4     | 4-7                       | 3     |
| 1-8                   | 1     | 1-8                       | 7     | 2-8                       | 6     | 3-8                       | 5     | 4-8                       | 4     |
| MD                    | 10/7  | MD                        | 4     | MD                        | 22/7  | MD                        | 18/7  | MD                        | 16/7  |

Figure 1. Axial maps, adjacency graphs and mean depth computation for two spatial systems, a grid and a simple tree. Complexity can be assessed by comparing the extreme values of mean depth within each system. It is shown that distributed spatial systems such as grids are more complex than non-distributed ones.

$$\text{COMPLEXITY INDEX} : \frac{\text{MD min}}{\text{MDmax}}$$

GRID 10/7 / 10/7 1.000

TRAIN 16/7 / 4 0.572

I given a fixed number of components, the most complex system is the one with the smallest number of sub-systems, that is, the one that could not be broken down into simpler systems by suppression of key components. To this extent, trees are very hierarchical systems and can be broken down into simpler sub-trees; grids are in the opposite situation;

II within distributed systems the relationship between each space and all others is much more complex than within non distributed ones, as far as the number of possible (shortest) paths between a pair of spaces is concerned;

III considering space as part of society (Soja, 1990) complex spaces are the ones which support the most complex pattern of social interaction. This is a very elusive condition, as “complex pattern of social interaction” can either be taken as the one that results from elaborate social rules, or the one that prompts elaborate social contact. The first case is the one of segmented societies where space, attaining apparent complexity, acts to control social interaction, preventing people placed in a certain area from interacting symmetrically with people from other places. The second is the one of less controlled societies in which space, apparently less complex, nevertheless allows everyone’s encounters with everyone.

Taking it dynamically, the system’s growth should also be captured as an increase in complexity. In this case individual RA values do not help to give a picture, as the mean depth is not always responsive to system growth. It would be possible to consider the system’s overall depth (the sum of all spaces’ depth) which should increase with the inclusion of any new component in the system. For two compared systems, one tree-shaped and the other grid-shaped, one would expect a pattern of linear growth in complexity as new components are added to the first system, whilst possible non linear growth for the second. Plotting the measures for two growing systems - a “train” and a “grid”- the resulting curves are contradictory, as illustrated in figure 2. Moreover, the manipulation of MD/RA indexes does not help to distinguish between two fundamental spatial situations - the urban and the suburban, the centre and the periphery - which could come into existence on similar street configurations. These are the cases of Manhattan compared to Los Angeles extensive suburbs, or the city of London compared to almost any English town suburb.

Complexity, within the context of Space Syntax, refers strictly to modes of grid articulation. The concept is associated to the number of possible alternative shortest paths between all pairs of spaces in the spatial system, being considered more complex than the system that does offer more alternative paths between any pair of destinations, hence being more open to social interaction. The extreme situations of complexity are, at the lower end, the “train”, which is a simplest tree-form system; at the higher end the perfect grid. Quantitative measures of complexity could be taken by directly counting the number of shortest paths of the entire system being studied and comparing it to those two extreme situations. Perfect grids present a very high correlation between the number of shortest paths and the number of islands (blocks), suggesting that the cyclomatic number (Kruger, 1979) could be used as an indicator of complexity.

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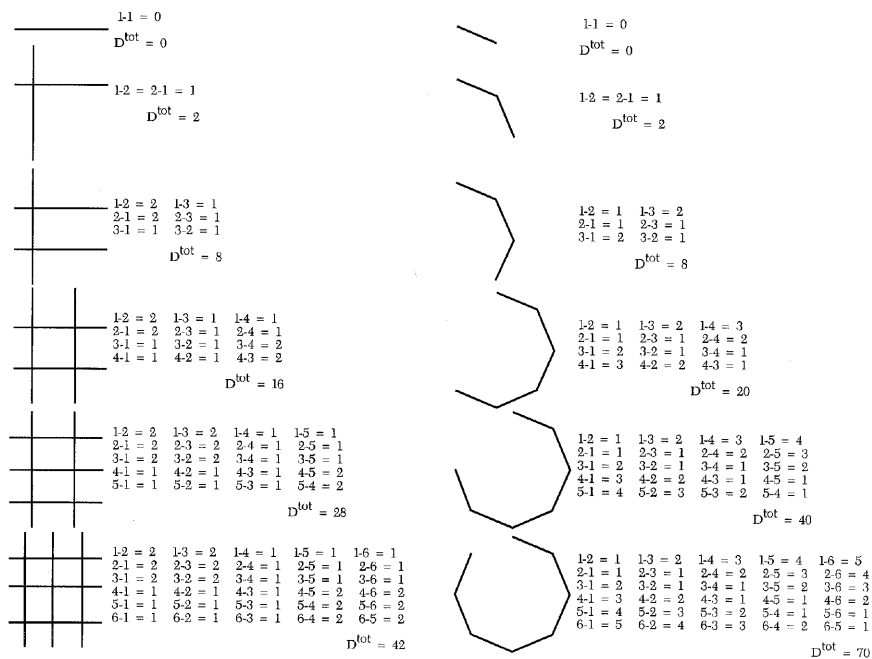


Figure 2. Assessment of complexity through measure of total depth. It is considered as two spatial systems that grow from one up to eight spaces. For both it is determined each space' depth and the system's total depth. Tree-form spatial systems sum up larger total depth measure than grid-form ones. The curves in the bottom show how complexity index grow along with the systems' growth.

### 3. Space, Centrality and Complexity

Another theoretic system for configurational modelling is Centrality (Krafta, 1994, 1996). Centrality is also a model of spatial differentiation, this time based on two spatial categories - public space and built form - and two spatial relationships - permeability and adjacency. According to this, the urban system is represented by a number of public spaces - axial lines, nodes or road links - and a number of built forms. Public spaces are related one to another by adjacency; built forms are related to public spaces by permeabilities and keep no relation to each other. Such a system can vary with the street layout as well as the distribution of built form units across it. As quantity of built forms matters, the system is able to distinguish between two different urban settlements based on similar street layouts, for example New York and Barcelona, or Barcelona and Los Angeles.

Centrality, as a measure of spatial differentiation, relies on the assumption that every built form unit is reachable from every other built form unit through the system of urban spaces. In this situation, the public spaces that provide such a reachability, that is to say, that fall on the shortest paths between any pair of built forms, are central to them. Every pair of built form units necessarily has a number of public spaces which are central to it - one space, in the case of both built forms being located on the same line, two, for the case in which built forms are located on adjacent lines, or more for other cases- falling on all alternative shortest path between them. If all pairs are processed, which implies to search for all possible shortest paths between each pair, to identify all spaces that fall on these shortest paths and to count how many times each space falls on the system's shortest paths, the public spaces can be ranked up according to their central role in providing reachability to all pairs of built form units. This rank is named betweenness centrality (Krafta, 1994).



The inclusion of a new variable - the built form unit - as well as new theoretical assumptions, brings about a new way to look at complexity. Firstly, the system description includes the built form, prompting a possible differentiation between scarcely/densely occupied grids. Secondly, in contrast to the RA model, which goes along a dimensionless accessibility logic, the centrality model comes near to the concept of spatial interaction. Spatial interaction (Wilson, 1987) evolves from early gravitational models which take the reciprocal attraction between two bodies, as a function of their masses and the distance separating them. Betweenness centrality, applied to urban spatial systems, can, in a similar way, be thought of as a process of sorting out attraction between pairs of elementary units of built forms. Instead of distance, number of spaces falling in the shortest path. Elementary units of built forms can be aggregated and expressed as attributes of the spaces they are permeable to, making the practical problem of centrality similar to spatial interaction, to process the relationship between pairs of spaces, each one loaded with a number of BF units and connected to each other through a number of other spaces. Spatial interaction is about flows between locations, generated by complementary activities; centrality is about tension between spaces, generated by uneven distribution of built form.

Complexity, in this case, becomes a matter of intensity, as well as distribution of tension. Speculation on Space Syntax has shown that distributed network systems add complexity; now it is necessary to assess the performance of built form distribution. It is not difficult to find out that, for a given system AB of A spaces and B built form units, a theoretically extreme concentrated built form distribution - all

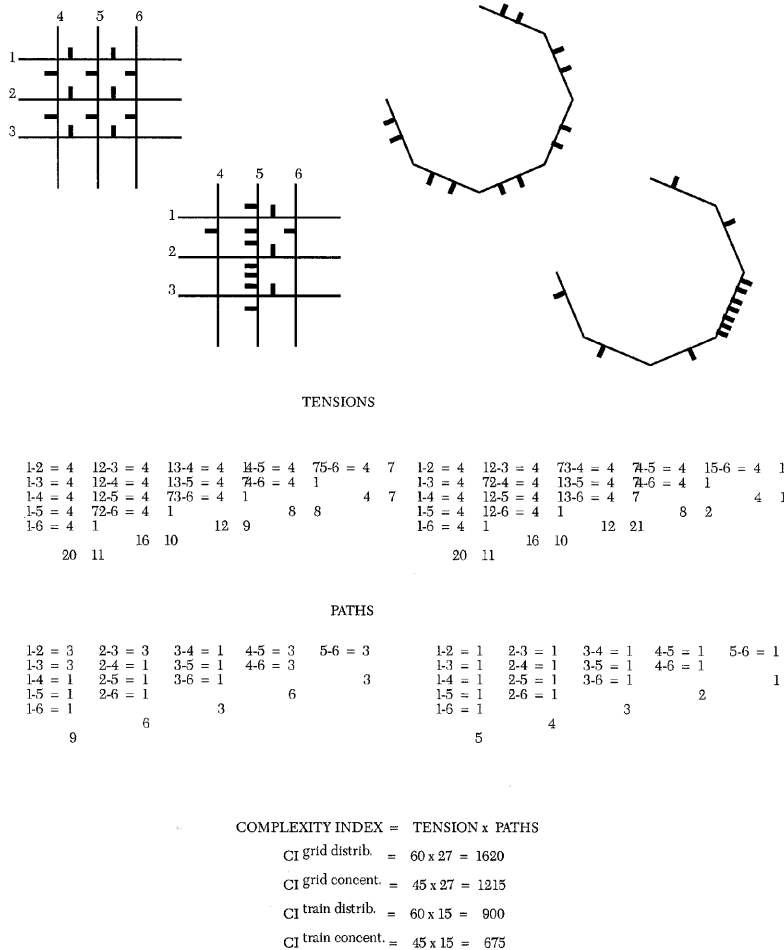


Figure 3. Assessment of complexity through paths and tensions. Four cases are examined: the first two are grid-form spatial systems, displaying alternative built form distribution; the other two cases are tree-form spatial systems, also holding different built form distribution. Results show that the model is responsive to changes in public space as well as built form distribution and articulation.

built form units located in only one space of the system - there would be no tension; on the other hand, for a perfectly distributed built form, tension will be maximum, regardless of the form assumed by the grid. To this extent, the system AB, referred to above, will see its complexity at the higher degree if and only if its street layout, as well as its built form are maximally distributed.

Progress can be made by looking at different combinations of public spaces and built forms, searching for the limits of complexity. First of all it is necessary to identify street layouts that give the maximum as well as the minimum number of shortest paths between pairs, considering the whole system. The lower extreme is easily identified as the “train”, which is the simplest version of a tree. From that it is possible to increase the linkage of the system by connecting its components, up to the perfect grid, which is the layout that reaches the peak, figure 3. By adding to the “train” a concentrated built form distribution, it is obtained the lowest possible complexity value and by adding to the perfect grid a perfectly distributed built form pattern, it is obtained the highest degree of complexity.

#### 4 Measurement of complexity

Consider a  $S$  system of  $A$  spaces and  $B$  built forms; the problem is to determine a measure of complexity as a function of amount of shortest paths and of tensions in the system. It is also required the definition of highest and lowest values of complexity, considering theoretical systems with the same amount of spaces and tensions.

##### 4.1. Path

A “train”, a tree-shaped spatial system, has only one path between any two spaces; hence the amount of paths is obtained by:

$$P(S)^{\min} = [A(A-1)] / 2 \quad (1)$$

In an orthogonal grid each space has  $A/2$  paths to each of the spaces parallel to it, and 1 path to each of the spaces orthogonal to it, so that:

$$P(S)^{\max} = [(A/2) - 1][A/2] + A/2 \quad (2)$$

Computing all spaces, the expression (2) comes to

$$P(S)^{\max} = A^3 / 8 \quad (3)$$

Between these two extremes, the system can assume any configuration and consequently any amount of paths. The solution to determine the particular amount of paths of a particular system seems to be algorithmic.

##### 4.2. Tensions

Regardless of the particularities of the street layout, distribution of built form units assumes its minimum value by concentrating stocks in just one space, resulting in a tensionless system. In effect, considering the stocks concentrated in the space  $i$ , all pairs that include  $i$  will have their tensions expressed by:

$$T(ij)^{\min} = T(i) \cdot T(j) = B \cdot 0 = 0 \quad (4)$$



Every pair that does not contain  $i$  will also be zero. On the contrary, perfect distribution of stocks will produce the maximum amount of tensions:

$$T(ij)^{\max} = T(i) \cdot T(j) = B/A \cdot B/A = [B/A]^2 \quad (5)$$

Taking all systems' pairs, the equation changes to:

$$T(S)^{\max} = [A(A-1)/2] [B/A]^2 \quad (6)$$

Similarly to paths, tensions can assume any configuration in particular systems, and its precise definition should be algorithmic.

### 4.3 Complexity

Taking complexity as a combination of both effects of grid configuration and built form distribution, the general expression for it is:

$$C(S) = P(S) \cdot T(S) \quad (7)$$

Once applied to the  $C^{\max}$  situation it comes to:

$$\begin{aligned} C(S)^{\max} &= P(S)^{\max} \cdot T(S)^{\max} \\ C(S)^{\max} &= [A^3/8] \cdot [A(A-1)/2] [B/A]^2 \end{aligned} \quad (8)$$

Considering that the lower end will always be zero, the problem of measuring complexity in relative terms resides in the construction of an algorithm that sorts out the exact amount of shortest paths between all pairs of spaces, processes the exact tension between each of them, calculates  $C(S)^{\max}$  and finally compares the results. The mathematical expression is

$$RC(S) = [P(S) \cdot T(S)] / C(S)^{\max} \quad (9)$$

The behaviour of the measure, considering a growing system should be examined. It is expected a non linear curve expressing the complexity values of a system which adds new spaces as well as built form units. Graphs in figure 4, below confirm such an expectation. In fact, both growth in spaces and built form units is non linear, even looked at separately.

*Figure 4. Graphs showing how the complexity index varies with changing spatial systems. The first pair of curves are for grid and tree-form systems in which spaces are kept unchanged and built form units increase. The second built form units are fixed and the number of spaces change. The third one shows the case of a full growing systems.*

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### 5 Concluding remarks

Complexity is a system measure, that is, it refers to the system as a whole and does not apply to each space belonging to the system in separate. According to the concept, explained in 3 and 4, above, the most complex city would assume the configuration of a perfect (topological) grid with perfectly distributed built form; a theoretical city that combines the Manhattan's grid (without Broadway) and Barcelona's evenly distributed built form. It is remarkable that apparently "complex" urban layouts, those which present complicated street design are not necessarily more complex than rather simple ones, such as the grids. Barcelona itself is one of these cases: the actual "diagonal" roads create straight links from each street to all its parallel in such a way that the number of minimal paths between each pair of parallel roads is reduced to just one, diminishing the overall amount of minimal paths. A similar, although less intense, effect on Manhattan is caused by Broadway.

Although complexity can be measured, and probably in many cases of large systems it must be measured, the concept is strong enough to allow qualitative assessment and interpretation. This seems to be a positive aspect of the theory, to the extent that designers can easily evaluate urban situations.

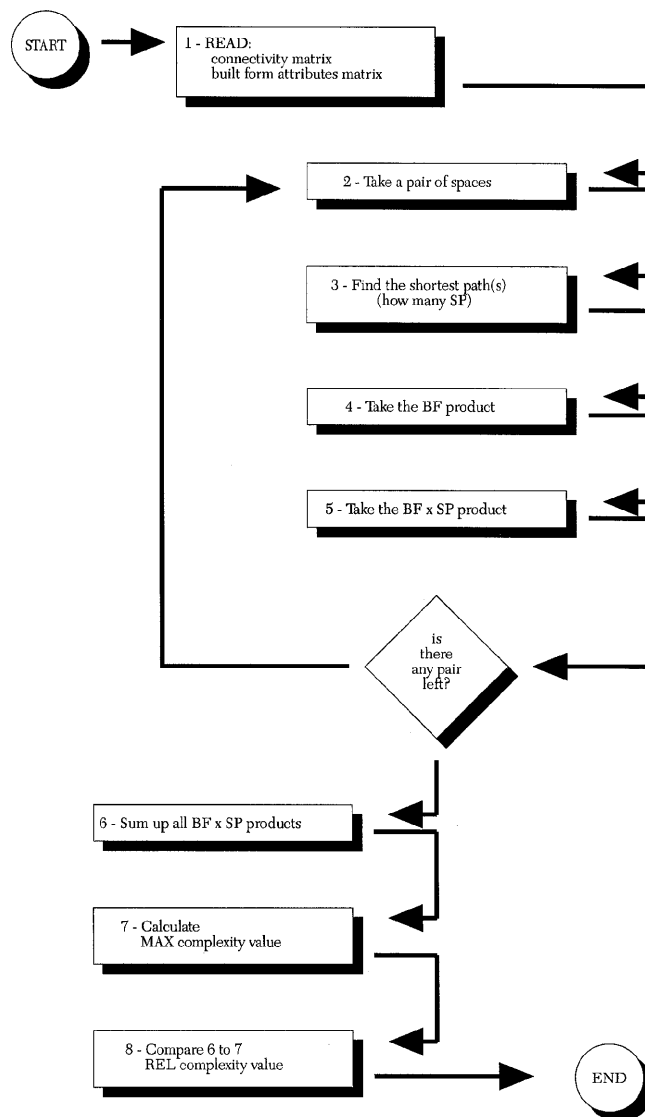


Figure 5. Flow chart for the algorithm that computes the measure of complexity

As mentioned in the introduction, urban spatial complexity is usually associated to other expressions, such as variety, scale (size), growth, intensity, continuity, density, both as a synthesis of them all, and as an individual expression of each one. The theory presented here seems able to articulate, in a precise way, things such as size, growth or density. It can be objected that variety does add an essential component to a true and realistic notion of complexity. In addition, it can be said that the model not only does not account for variety, but also that grids and homogeneous built form distribution do not actually fit the very notion of variety. The conceptual objection is comparable to the one, already examined, that deals with the dualism of open society/complex space. In fact, variety is expressed in terms of collections of formally different built forms, original street layouts and articulation between both. It has already been shown that the capacity of grids as generators of typological diversity, along all the above quoted dimensions (Martin, 1972). Moreover, when perfect grids are referred to in the text, this does not necessarily mean a geometric perfection, but just a topological one. Perfect grids can be as deformed as the ground determines and the designer or the developer wishes without losing their typological perfection.

The instrumental objection about the model's inability to cope with variety is substantive in some sense, and prompts an interesting possible unfolding. It has been said here that in practise, built form is computed in aggregated form according to its distribution across the system of spaces. In this sense, built form quantities enter the model as attributes to axial lines, links or whatever space unit is adopted. Attributes can be specified further, so that built form can enter the model as different quantities of various built form types. The total quantities do not change nor does the primitive measure of complexity; however, new possibilities are introduced in the model with built form qualification. Consider, as an example, a system in which three built form types (red, blue and yellow) are included. As a result of a random distribution, there would be six possible types of tension: red, blue and yellow, resulting from pairs of same colour, as well as green, orange and purple tensions arising from pairs of different BF types. Imagine these tensions as coloured lines in the map; spaces would be crossed over by bunches of lines of different colours, denouncing each space's role in providing reachability to different types of built form units. Consider, moreover, that built form types are, in a sense, related to activities, the complexity map would show not only the system's degree of complexity, but also a sort of inner qualification of spaces. Algorithmically there is no major obstacle to do that, apart from possible increase in processing time.

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